RETURNS AND VOLATILITY:

A STUDY OF THE IRISH EQUITY MARKET

TAT Y. CHOI AND LOUIS MURRAY

University College Dublin

ABSTRACT

In an important advance, Merton (1980) proposed that the development of reliable variance estimation models would play an important part in the study of price volatility. In practice, this implies that pricing models should take account of heteroscedasticity. The development of ARCH models (Engle, 1982) is important, as they do this by specifically defining the changing conditional variance. They have had a tremendous impact on financial research.

This paper examines the relation between stock returns and volatility, using Irish market data. The two basic hypotheses are: (1) that expected returns are positively related to expected volatility (a higher risk premium would be required in compensation for higher risk exposure), and (2) unexpected returns are purported to be negatively related to unexpected volatility (a higher than average return may cause a decrease in volatility).

Market volatility is examined, using ARMA models. They allow the estimation of expected volatility. Conditional variances are also generated, using a number of variants of the ARCH models. The significance of different possible relationships can then be tested.

INTRODUCTION

The vast majority of empirical work in finance has been based on the underlying assumption of an efficient market hypothesis. Following this, a number of static asset pricing models are employed. The Capital Asset Pricing Model, for example, implies that expected returns on shares are a positive linear function of their market betas, and also that market betas suffice to describe the cross-section of expected returns. Other variants of this model take a similar form.

A large amount of research has been conducted on share price data, using models which require the assumption of an efficient capital market. Studies have been conducted on both large and small stock exchanges.

The Dublin exchange is no exception, and a number of studies have been carried out on Irish data, with mixed results. It is likely, however, that the small size of the Dublin market will result in inefficiencies, casting some doubt over the appropriateness of models of this type.

Further, a sizeable amount of evidence has been appearing in the academic literature, which indicates the existence of equity return regularities (for example, a size effect, a January effect, a p/e ratio effect, etc). These results clearly contradict an efficient market hypothesis, as they indicate the possibility of gains based on specific trading rules. This should not be possible within the environment of an efficient market.

In fact, Fama and French (1992) show, using American data, that two easily measurable variables, size and the ratio of book value to the market value of equity, actually provide a better characterisation of average returns than does beta, the normal measure of risk. Although their results have been widely questioned, there is an implication that alternative equilibrium models of equity returns may be more appropriate.

The academic community has, as a result, been turning its attention to alternative models of share price returns. These models search for the existence of identifiable factors which may contribute to the explanation of movements of equity returns. There is considerable interest in models which take account of price volatility. Merton (1980) proposes that estimators which use realised returns must take account of heteroscedasticity. The implication is that a more appropriate model will take account of errors in the variance estimates.

The development of ARCH models (Engle, 1982) makes this possible. They provide a direct test of the relationship between equity returns and volatility. They also allow for a changing conditional variance.

Models of this type were initially fitted to inflation rates (Engle, 1982) and currency exchange rates (Hsieh, 1989). They have also been applied to equity returns data in the US and the UK. Using NYSE data, French, Schwert and Stambaugh (1987) find that the expected market risk premium is positively related to the predictable volatility of stock returns. They also find evidence that unexpected market returns are negatively related to volatility. Using similar data, Baillie and DeGennaro (1990) find weak evidence of a relationship between expected returns and volatility. Using daily, weekly, fortnightly, and monthly returns on the FT All Share Index from 1965 to 1989, Poon and Taylor (1992) find a significant relationship between returns and volatility, only when volatility is represented by standard deviation.

As these models allow investigation of the relationship between returns and volatility, a number of hypotheses may be tested. It can be argued, for example, following the experiences of the 1987 crash, that lower than expected returns will cause an increase in speculative activity. This would imply a negative relationship. The relationship between expected returns and volatility may also be tested. A positive relationship may be expected, as a higher risk premium would provide compensation for risk, when volatility is high.

Two stages are required for the investigation of these relationships. Firstly, daily market returns are used to compute monthly estimates of volatility. These estimates are then decomposed into their predictable and unpredictable components, using Autoregressive Integrated Moving Average Models. The relationship between monthly returns, and both the predictable and unpredictable elements of volatility, can then be investigated.

Daily returns are also used to provide ex-ante measures of volatility, using Generalised Autoregressive Conditional Heteroscedasticity Models (Bollerslev, 1986). They recognise the presence of successive periods of relative volatility and stability, and they allow the conditional variance to evolve over time, as a function of past errors and past variances.

PROPERTIES OF THE DATA

The data used in this study was collected from the Dublin Stock Exchange. It consists of daily values of the ISEQ all share value weighted index, covering the full market and the unlisted securities market. The period covered is January 1987 to July 1993. Only weekly index values are available for the period to 1987, so they are excluded from the study.

Daily market return is computed using the following formula:

(1) $R_t = Ln(P_t) - Ln(P_{t-1})$

where P_t is the index value on day t, and t-1 represents the previous day. The index series is not adjusted for dividends, so there could be a systematic bias in the return distributions, as ex-dividend days will tend to fall on particular days of the week and at particular times of the year. The impact should be small, however, and Poon and Taylor (1992) show, using UK data, that there is a very high correlation between dividend adjusted and unadjusted market returns. The exclusion of dividend data has a minimal impact on index return distributions.

Monthly variance of market returns is then calculated. Non-overlapping samples of daily returns are used to prepare the monthly estimates. This allows a more precise estimation of the variance for a particular month, as the volatility of stock returns may not be consistent from month to month. Also, as volatility is estimated using very frequent measures of return, the accuracy of estimates should be improved.

The standard deviation of monthly return is estimated as follows (see French et al., 1987):

(2) $$\sigma_{mt} = [\sum_{i=1}^{N_t} R^2_{it} + 2\sum_{i=1}^{N_{t-1}} R_{i+1t}]^{1/2} \ldots \Sigma i=1 \ldots N_t, N_{t-1}$$

where there are N_t daily returns (R_{it}) in month t. As thin trading can cause index returns to be autocorrelated, variance is initially estimated as the sum of squared daily returns plus twice the sum of the products of adjacent returns. Standard deviation is then directly measured from this estimate of variance. As the Dublin market certainly is subject to thin trading (Murray, 1992), such an adjustment should improve the quality of estimates.

Further, it is the normal practice to subtract the within month mean from each observation, but this adjustment is very small, and as Merton (1980) notes, it will have little effect on the results. It is therefore excluded.

The monthly standard deviation estimates are plotted in **Figure 1**. It is noticeable that the pattern of volatility is not stable. Considerable increases can be identified in both late 1987 and 1990. It is entirely possible therefore that these variations may explain the varying levels of return experienced on the Irish equity market.

Details of the monthly estimates of standard deviation are also presented in **Table 1**. The autocorrelations are large, but they do decay rapidly after lag three. The estimates also are highly skewed. In order to adjust for skewness, the series is transformed by taking the natural logarithms ($Ln(\sigma_{mt})$). Details of this series are also presented in **Table 1**. The

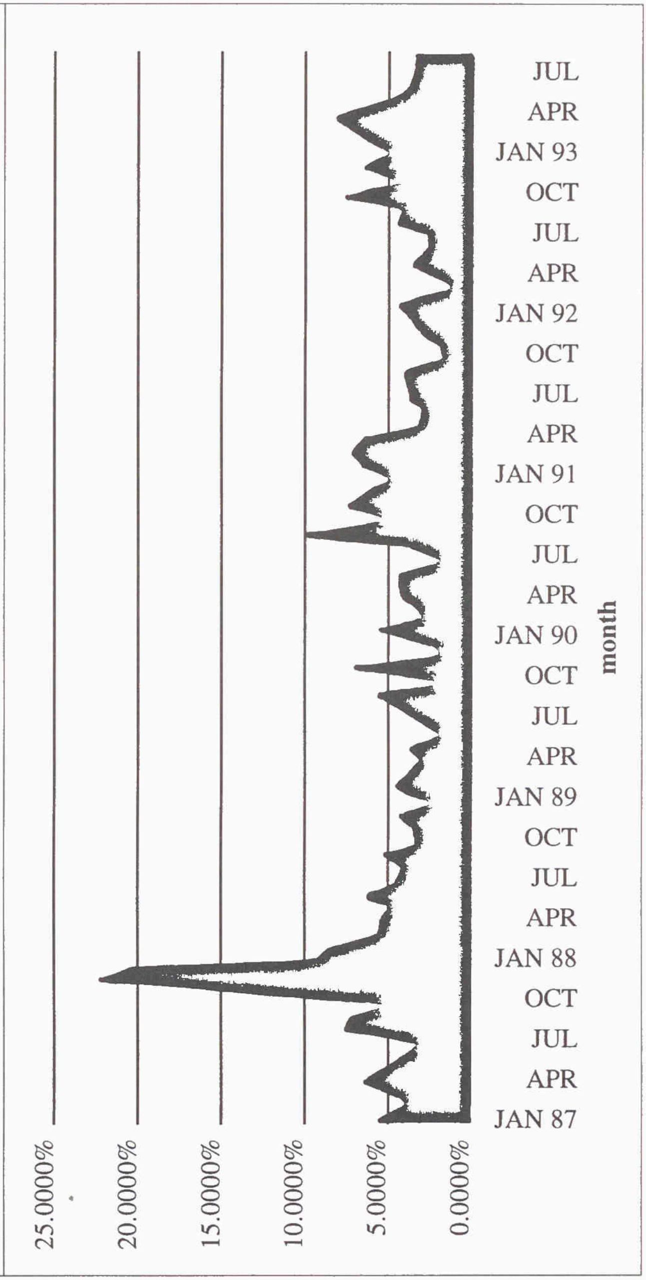


Figure 1: ISEQ INDEX
Monthly Standard Deviation – 01/1987 – 07/1993

autocorrelations of this series are substantially large, and they decay steadily until lag ten. The impact of skewness is effectively removed, however.

PREDICTABLE AND UNPREDICTABLE VOLATILITY

The autocorrelation coefficients of $Ln(\sigma_{mt})$ reported in **Table 1** indicate a slowly decaying autoregressive effect. This suggests that the first differences of the log of standard deviation may follow a moving average process of more than three orders. The results could even indicate that non-stationarity is present in the process of volatility.

In order to further explore the autocorrelation characteristics of the volatility series, a range of Autoregressive Moving Average Models (ARMA) have been fitted to the log transformed data. The appropriate number of moving averages is estimated by increasing, one lag at a time, up to seven, the number of moving averages in the ARMA process:

(3) $MA(1): (1- \Phi B)Ln(\sigma_{mt}) = \theta_0 + (1 - \theta_1 L)U_t$

$MA(2): (1 - \Phi B)Ln(\sigma_{mt}) = \theta_0 + (\theta_1 L - \theta_2 L^2)U_t$

.

.

$MA(7): (1 - \Phi B)Ln(\sigma_{mt}) = \theta_0 + (1 - \theta_1 L - \theta_2 L^2 ... \theta_7 L^7)U_t$

where Φ represents the autoregressive term and θ represents the moving average terms.

Estimates of the coefficients of these models are presented in **Table 2**. Estimates of the constant term (θ_0) remain small, in relation to their standard errors, regardless of which form of an ARMA model is used. Coefficients of the MA terms (θ_1, θ_2, etc) are sizeable if up to three moving averages are fitted to the data. The inclusion of further lags does increase the R^2 values; however, a lack of significance in coefficients of the MA terms does indicate that considerable noise is being added to the estimation process. It is concluded therefore that no further benefit can be derived by fitting further MA terms to the ARMA model. An ARMA(1,3) model is therefore used to estimate the predicted and unpredicted elements of the volatility process. This is very similar to the IMA(1,3) model which French et al. recommend as being the most suitable to apply to US data.

Table 1: Estimates of The Standard Deviation of the Return to ISEQ (1987-1993)

		Autocorrelation at Lags													
Mean	St.Dev.	1	2	3	4	5	6	7	8	9	10	11	12	Std. Er.	Q (12)
a. Monthly Standard Deviation of ISEQ estimated from Daily Data															
.0508	.0326	.49	.28	.27	.10	-.05	.08	.09	-.05	-.02	.00	-.12	-.06	0.11	36.50
b. Ln of Monthly Standard Deviation of ISEQ Returns estimated from Daily Data															
-3.11	.5197	.35	.30	.34	.22	.19	.18	.09	-.06	.05	-.02	-.19	-.13	0.11	43.55

Note: Q(12) is the Box-Pierce-Ljung Statistic for 12 lags of Autocorrelation

Table 2: ARMA Models

Ln of the Monthly Standard Deviation of ISEQ Returns estimated from Daily Data (1987 - 1993)

Lags MA (X) X=1..7	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	R^2	$S(U_t)$	Skewness	$SR(U_t)$	F-Test	Q(12)
1	-.5251	.5684							.1859	.4657	-.04002	6.3678	11.6532	8.88
	(.3814)	(.184)												
2	-.5762	.0601	.0761						.1892	.4647	-.01744	6.2886	5.8293	8.53
	(.4517)	(.182)	(.133)											
3	-.7582	.5283	.0117	-.158					.2045	.4602	-.04987	6.3010	4.0662	5.94
	(.5626)	(.204)	(.135)	(.131)										
4	-.7381	.5344	.0130	-.163	.0131				.2047	.4602	-.06447	6.2933	3.0117	5.90
	(.7520)	(.269)	(.139)	(.134)	(.156)									
5	-.9139	-.494	-.023	-.168	-.066	-.131			.2117	.4582	-.02316	6.1776	2.4364	4.99
	(.8186)	(.276)	(.140)	(.131)	(.143)	(.132)								
6	-1.3798	.3380	-.054	-.180	.0311	-.142	-.104		.2161	.4547	-.08956	6.2690	2.0328	4.42
	(1.146)	(.375)	(.136)	(.133)	(.143)	(.123)	(.152)							
7	-4.3186	-.622	-.237	-.303	-.238	-.235	-.265	-.240	.2228	.4543	-.04042	6.3385	1.7563	3.43
	(1.641)	(.509)	(.170)	(.155)	(.166)	(.138)	(.146)	(.122)						

Note: Q(12) is the Box-Pierce-Ljung Statistic for 12 lags of autocorrelation
$SR(U_t)$ is the student range
$S(U_t)$ is the standard deviation of error terms

The results from fitting an ARMA(1,3) model to the data are summarised in **Table 2**. As noted above, the moving average estimate at lag one is significant. Moving average estimates at lags two and three are sizeable; however, they do not differ significantly from zero. Nonetheless, by using an MA model at lags two and three, the overall fit is improved. A potential limitation must be recognised, however, as all of the predictions are in sample.

Conditional forecasts of the standard deviation and variance of returns can be prepared, using estimates of the ARMA(1,3) model. Standard deviation (δ_{mt}) is forecast as:

(4) $\delta_{mt} = \exp[Ln(\sigma_{mt}) + 0.5V(U_t)]$

and variance is:

(5) $\delta^2 = \exp[2Ln(\sigma_{mt}) + 2V(U_t)]$

where $Ln(\sigma_{mt})$ is the fitted value, and $V(U_t)$ is the variance of the prediction errors (U_t). These forecasts require an assumption that the prediction errors are normally distributed, and that σ_{mt} is lognormal. As can be expected, forecast standard deviation is very similar to actual standard deviation, although it is smoother. The forecast standard deviations are plotted in **Figure 2**. The pattern is similar to actual volatility, as presented in **Figure 1**.

These forecast values can be treated as expected or predictable volatility of returns that should be known to investors. Using these results, unexpected volatility (σ_{mt}^u) can therefore be defined as:

(6) $\sigma_{mt}^u = \sigma_{mt} - \delta_{mt}$ (using standard deviations)

or:

(7) $\sigma^2{}_{mt}^u = \sigma^2_{mt} - \delta^2_{mt}$ (using variances)

Expected and unexpected volatility values are summarised in **Table 3**. There is little evidence of skewness. There is, however, some evidence of autocorrelation in the two predicted volatility series, particularly if standard deviation is used as the measure. An ARMA process should fully allow for such a pattern, but the levels of autocorrelation are considerably reduced, in comparison with the original series and the log transformed series. Poon and Taylor (1992), using UK data, also

Figure 2: ISEQ INDEX Predicated Monthly Standard Deviation 01/1987 – 07/1993, using ARMA model

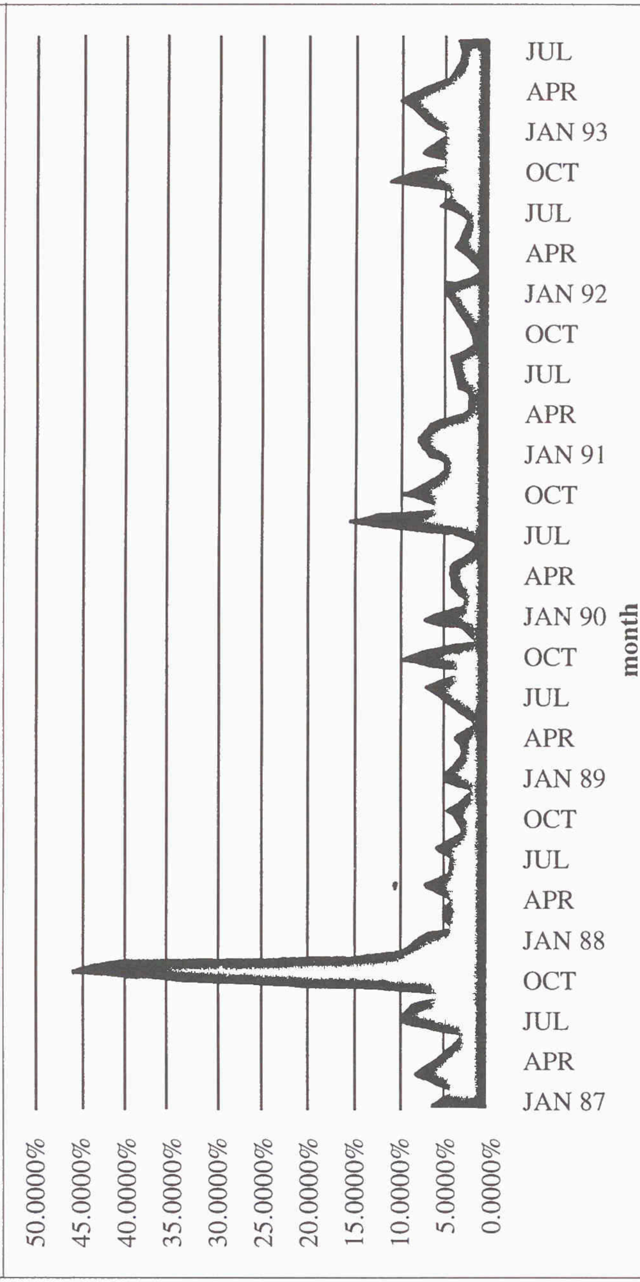


Table 3: Estimates of Predicted and Unpredicted Volatility using ARMA

	N	Mean	STD. Dev.	Min.	Max.
σ_{mt}^{u}	79	-.00129	.00536	-.03780	.00270
$\sigma^{2}{}_{mt}^{u}$	79	-.00109	.00549	-.04333	.00052
δ_{mt}	79	.05211	.03762	.00807	.26065
δ^{2}_{mt}	79	.00472	.01252	.00005	.00929

Auto-Correlations at lags

	1	2	3	4	5	6	7	8
σ_{mt}^{u}	.35	.03	.02	-.09	-.09	.04	.03	-.07
$\sigma^{2}{}_{mt}^{u}$	.39	-.01	-.01	-.04	-.05	-.04	-.02	-.04
δ_{mt}	.47	.24	.23	.07	.03	.06	.08	-.05
δ^{2}_{mt}	.47	.07	.06	-.02	-.04	-.02	.01	-.04

report evidence of autocorrelations in the fitted values.

The patterns of autocorrelation in unpredicted volatility are very slight. Some evidence of autocorrelation can be expected, as this series is not directly generated from an ARMA fitted series.

A further test is to examine the relationship between returns and volatility. In an efficient market, investors may consider the best available predictions of standard deviation or variance that will affect the equilibrium of returns on the market. Levels of predicted volatility may therefore have an impact on market returns.

A better relationship may be found if account can be taken of the risk premium on equity (R_{mt} - R_{ft}), where R_{ft} refers to the risk free rate of interest. The equity risk premium, rather than overall return on the equity markets, may react more accurately to levels of volatility, either predicted or unpredicted. This improvement will allow for any movements in the underlying rates of interest throughout the study period. There is no obvious reason why interest rate movements should be related to levels of volatility on the equity markets.

The yield on a ninety-one day Government Exchequer Bill has been used as the measure of a risk free rate of return on the Dublin market. Shorter dated bills have been traded, but a continuous series of quotes was not available throughout the study period. The longer dated bill should provide an acceptable and consistent measure of risk-free return, assuming that the shape of the yield curve remains reasonably constant.

The relationship between return and volatility is therefore tested as follows:

(8) $R_t = \alpha + \beta Vol_{mt} + \varepsilon_t$

where R_t: either R_{mt}, or (R_{mt} - R_{ft})

Vol_{mt}: Either predicted, or unpredicted volatility

also, the relationship is tested, using both variance and standard deviation as the measure of volatility.

Results of the regression tests are presented in **Table 4**. Using either measure of volatility, it is clear that there is a strong and significant negative relationship between predicted volatility and return. The t-statistics indicate greater significance when variance is used. The R^2 statistics also are higher. Also, the relationship is significant, regardless of whether return on the market index (R_{mt}), or risk premium (R_{mt} - R_{ft}), is used as the measure of return.

This is the opposite to the relationship proposed by French, Schwert and Stambaugh (1987). It should be noted, however, that using American data, they were unable to find evidence of a significant relationship, in either direction.

A possible explanation for this result is that persistent thin trading causes a delay in the market reaction to increases in volatility. If the delay

Table 4: Regression Analysis of Relationship between Returns and Estimates of Volatility, using an ARMA model

1 $R_{mt} = \alpha + \beta\delta_{mt} + \varepsilon_t$

2 $(R_{mt} - R_{ft}) = \alpha + \beta\delta_{mt} + \varepsilon_t$

	1) α	β	R^2	2) α	β	R^2
Std. Dev.	.6062	-10.3	.1998	.5052	-10.3	.1979
	(3.97)	(-4.3)		(3.31)	(-4.3)	
Variance	.2652	-42.4	.3672	.1644	-42.3	.3649
	(3.31)	(-6.6)		(1.91)	(-6.36)	

3 $R_{mt} = \alpha + \beta\sigma_{mt}^{u} + \varepsilon_t$

4 $(R_{mt} - R_{ft}) = \alpha + \beta\sigma_{mt}^{u} + \varepsilon_t$

	3) α	β	R^2	4) α	β	R^2
Std. Dev.	.1901	96.7	.3507	.0898	96.7	.3500
	(2.31)	(6.45)		(1.09)	(6.44)	
Variance	.1764	102.2	.4114	.0758	102.0	.4089
	(2.27)	(7.34)		(0.97)	(7.29)	

5 $R_{mt} = \alpha + \beta\delta_{mt} + Y\sigma_{mt}^{u} + \varepsilon_t$

6 $(R_{mt} - R_{ft}) = \alpha + \beta\delta_{mt} + Y\sigma_{mt}^{u} + \varepsilon_t$

	5) α	β	Y	R^2
Std. Dev.	-.668	19.68	225.7	.4414
	(-2.6)	(3.51)	(5.74)	
Variance	.0221	67.83	254.6	.4371
	(0.19)	(1.83)	(3.07)	

	6) α	β	Y	R^2
Std. Dev.	-.772	19.77	226.3	.4414
	(-3.0)	(3.52)	(5.75)	
Variance	-.078	67.95	254.6	.4347
	(-.69)	(1.86)	(3.06)	

were consistent, which might be expected if levels of thin trading were constant throughout the period, it could result in a negative relationship between market return and volatility.

In order to test this proposition, the volatility measures are regressed on returns for the following period (R_{t+1}). No significant relationship could be found; however, the coefficients are positive, suggesting the expected relationship. The conclusion that thin trading may be the reason for an unexpected negative relationship cannot therefore be confirmed, but it remains a strong possibility.

Following the negative relationship between contemporaneous returns and expected volatility, a positive relationship is identified between returns and unexpected volatility. The relationship is strongly significant, regardless of which measure of returns or volatility is used. The R^2 values are particularly high, if variance is used.

As a final test, the measures of both expected and unexpected volatility are regressed on contemporaneous market returns:

(9) $R_t = \alpha + \beta\delta_{mt} + Y\sigma_{mt}^{u}$ (standard deviations)

and:

(10) $R_t = \alpha + \beta\delta^2_{mt} + Y\sigma^2_{mt}{}^{u}$ (variances)

As unexpected volatility should not be correlated with the expected element of volatility, its inclusion should not affect the quality of estimated coefficients. Some caution is necessary, however, as theoretically there could be a positive relationship between expected returns and expected volatility, and also between unexpected returns and unexpected volatility. As raw returns can be presumed to be a good proxy for unexpected returns, and a less suitable one for expected returns, it is possible that the measure of unexpected volatility will dominate this causal relationship.

Coefficients for both expected and unexpected volatility are positive in this instance, indicating the anticipated relationship with market return, or risk premium. The t-statistics indicate that unexpected volatility is the more significant predictive variable. Some caution is necessary in interpreting this result, however. As noted above, the relationship between raw returns and unexpected volatility may dominate.

THE ARCH MODEL

Engle's (1982) autoregressive conditional heteroscedasticity (ARCH) model is used to generate a series of changing volatility. Essentially, an ARCH suggests that in a time series, the large and small forecast errors appear to occur in clusters. The variance of the forecast errors thus depends on the size of the preceding disturbance.

A single version of an ARCH process is as follows:

(11a) $R_t = \alpha + \varepsilon_t;\ \varepsilon_t \sim N(0,h^2_t)$

(11b) $h^2_t = a + a_1\varepsilon^2_{t-1}$

It assumes that volatility is a deterministic function of past returns, that is, h^2_t is conditional on ε^2_{t-1} at time t-1. Daily risk premium is derived using the following formula:

(12) $R_{pt} = R_{it} - R_{ft}$

where R_{it} refers to daily return on the ISEQ index, and R_{ft} is the average daily yield on the ninety-one day Government Exchequer Bill, issued at the beginning of each month.

Since there are no daily issues of any government instruments in Ireland, it is necessary to translate the monthly yield on the ninety-one day bill into an average daily yield for the entire month. Although this measure does not account for rate changes within a particular month, it is the most complete measure available. For the few months around the ERM crisis of 1992/93, the government did not issue ninety-one day bills. The yield on a thirty-five day bill was used over this period.

As the equity market will be subject to thin trading, the ARCH model is generalised by taking account of a first order moving average process. Equation (11a) therefore becomes:

(13) $(R_{it} - R_{ft}) = \alpha + \varepsilon_t - \theta_{t-1}$

where $R_{it} - R_{ft}$ is the daily excess holding period return on the ISEQ index, and θ is the moving average coefficient, which is expected to be negative.

Conditional volatility can be defined using either a simple ARCH form, or the Generalised Autoregressive Conditional Heteroscedasticity form (GARCH), as proposed by Bollerslev (1986). Both forms are shown below:

(14) $h^2_t = a + a_1\varepsilon^2_{t-1}$ (ARCH)

(15) $h^2_t = a + a_1h^2_{t-1} + b_1\varepsilon^2_{t-1} + b_2\varepsilon^2_{t-2}$ (GARCH)

Table 5 sets out the estimates of the ARCH and GARCH models for 1987-1993.

The ARCH model estimate for a_1 is 0.42, with a t-test of 7.944 and a standard error 0.053. This demonstrates that there clearly is a relationship between the variance (h^2_t), and the recent squared error (ε^2_{t-1}). The estimate for θ, which is intended to capture the effect of non-synchronous trading, is negative, as expected; however, the t-test is less than significant. The standard error for θ is 0.019.

Results from fitting the GARCH model are even more impressive. First, the sum of ($a_1 + b_1 + b_2$) is less than one, indicating that the volatility process is stationary (Bollerslev, 1986). Secondly, the t-test for a, a_1, b_1, and b_2 are all within a reasonably high degree of significance. In addition, the θ coefficient again is negative, although the t-test is not significant.

GARCH-IN-MEAN

Following French, Schwert and Stambaugh (1987), Engle, Lilien and Robins (1987), and Bollerslev, Engle and Wooldridge (1988), estimates of the GARCH-IN-MEAN function are generated. The GARCH-IN-MEAN allows conditional return to be a function of volatility. Hence, using this concept, models in the following forms can be introduced:

(16a) $(R_{it} - R_{ft}) = \alpha + \beta h_t + \varepsilon_t - \theta\varepsilon_{t-1}$

(16b) $(R_{it} - R_{ft}) = \alpha + \beta h^2_t + \varepsilon_t - \theta\varepsilon_{t-1}$

(16c) $h^2_t = a + bh^2_{t-1} + c_1\varepsilon^2_{t-1} + c_2\varepsilon^2_{t-2}$

As defined earlier, $R_{it} - R_{ft}$ is the daily excess holding period return on the ISEQ index.

Table 5: Autoregressive Conditional Heteroscedasticity (ARCH) for Daily Excess Holding Period Returns to ISEQ

ARCH: $(R_{it} - R_{ft}) = \alpha + \varepsilon_t - \theta\varepsilon_{t-1}$; $h^2_t = a + a_1\varepsilon^2_{t-1}$

GARCH: $(R_{it} - R_{ft}) = \alpha + \varepsilon_t - \theta\varepsilon_{t-1}$; $h^2_t = a + a_1 h^2_{t-1} + b_1\varepsilon^2_{t-1} + b_2\varepsilon^2_{t-2}$

	a	a_1	b_1	b_2	α	θ
ARCH	$.778X10^{-4}$	.425			$-.224X10^{-3}$	$-.10X10^{-1}$
	(21.944)	(7.944)			(-.954)	(-.513)
GARCH	$.106X10^{-4}$	.214	.298	.402	$.135X10^{-3}$	-.0358
	(5.291)	(6.59)	(2.52)	(3.84)	$(.136X10^{-3})$	(-1.02)

The value in parenthesis is the t-test statistic.

Equation (16a) considers conditional return in terms of standard deviation, while equation (16b) is in terms of variance. And h^2_t is the variance of the unexpected excess holding period return (ε_t), and ε_{t-1} is again used to capture the effect of non-synchronous trading.

Table 6 establishes the GARCH-IN-MEAN investigation. It can be noted that α is the same, regardless of whether conditional return is defined in terms of standard deviation or variance. If we believe that α reflects the dimension of an average daily risk premium, this daily risk premium can be translated into an annual risk premium of approximately 12%.

The results in **Table 6** are, to some extent, inconclusive. When a variance specification is used β is positive, while it becomes negative if the standard deviation specification is applied. Both β values are small, however, indicating that the effect will be insignificant. The results shown here are, to some extent, consistent with Poon and Taylor (1992). They identified an insignificant positive relationship between share returns and volatility expectations, as represented by variance. But, when using standard deviation as the measure of volatility, their results do not coincide, as they again found a small positive relationship.

In order to be able to compare the GARCH-IN-MEAN models and the results from fitting an ARMA model, a series of monthly predicted variances is produced from the daily variances generated in the GARCH-IN-MEAN model. Monthly predicted variance is derived as follows:

(17) $\eta^2_{mt} = (\Sigma h^2_{it})/N_t; \qquad \Sigma i = 1....N_t$

where N_t represents the number of trading days in month t.

Figure 3 shows the monthly standard deviation, as predicted by the GARCH-IN-MEAN model. As can be seen, the movement of predicted standard deviation (δ_{mt}) is similar to the ARMA prediction (η_{mt}), although the coefficient of the GARCH-IN-MEAN prediction is much lower than the ARMA model.

CONCLUSIONS

Two statistical approaches have been employed to test the relationship between equity returns and volatility on the Dublin market.

Using an ARMA model, overall volatility was decomposed into two

Table 6: GARCH-IN-MEAN Models for Daily Excess Holding Period Returns to ISEQ Index (1987-1993)

Standard Deviation: $(R_{it} - R_{ft}) = \alpha + \beta h_t + \varepsilon_t - \theta\varepsilon_{t-1}$

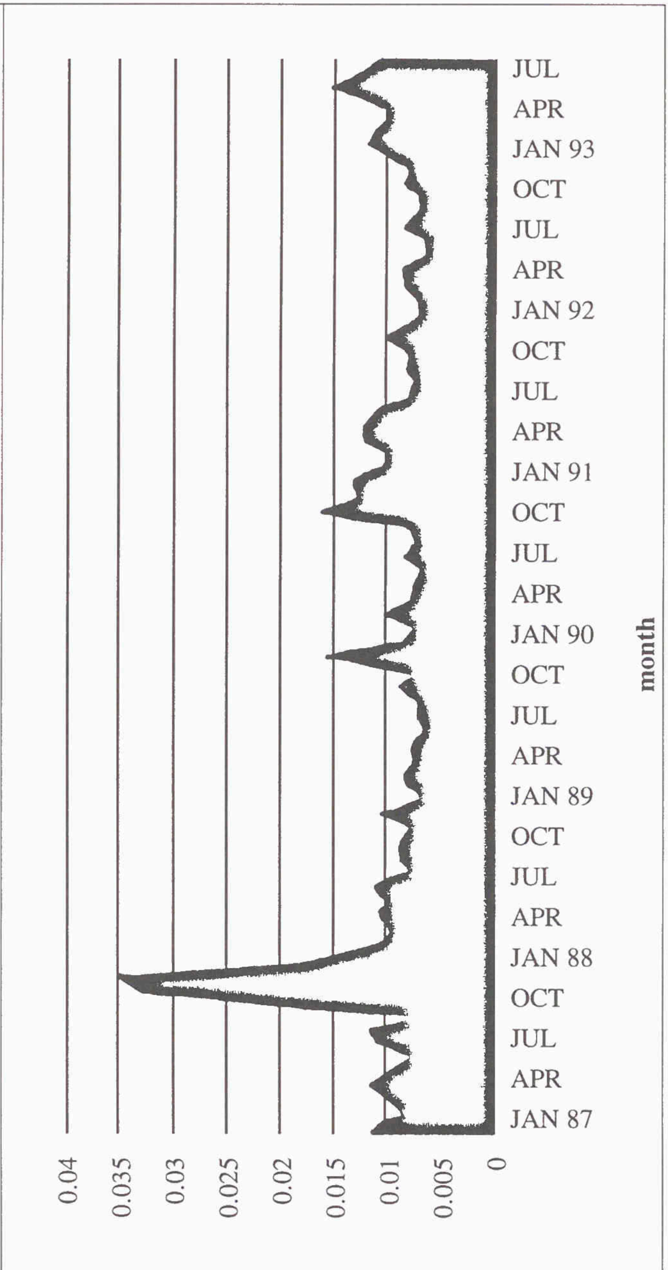
Variance: $(R_{it} - R_{ft}) = \alpha + \beta h^2_t + \varepsilon_t - \theta\varepsilon_{t-1}$

$$h^2_t = a + bh^2_{t-1} + c_1\varepsilon^2_{t-1} + c_2\varepsilon^2_{t-2}$$

	α	β	c_1	c_2	a	b	θ
Std Dev	.446X10	-.154X10	.213	-.166	.544X10	.849	-.504
	($.212X10^{-18}$)	($.186X10^{-16}$)	($.12X10^{-2}$)	($.179X10^{-6}$)	($.812X10^{-6}$)	($.465X10^{-2}$)	($.843X10^{-17}$)
Variance	.446X10	.363X10	.213	-.166	.544X10	.849	.319
	($.188X10^{-18}$)	($.696X10^{-15}$)	($.12X10^{-2}$)	($.179X10^{-2}$)	($.812x10^{-6}$)	($.465x10^{-2}$)	($.690x10^{-15}$)

The value in parenthesis is the standard error; the t-tests for β (variance) is -8.238 and β (std. dev.) is 5.2146

Figure 3: ISEQ INDEX
Predicted Monthly Statement Deviaton 01/1987 – 04/1993, Using Garch-in-mean Model



elements, predicted and unpredicted volatility. The results were inconclusive. There is some evidence of a negative relationship between predicted return and volatility. This is unexpected. Further testing suggests that this may be due to thin trading, but the results again are inconclusive. Evidence of a positive relationship between unpredicted volatility and return has also been found.

Using a GARCH model, the relationship between volatility expectations and returns was examined. An insignificant positive relationship is identified if variance is used; however, the relationship becomes negative when a standard deviation specification is employed.

A comparison between actual monthly volatility and predicted volatility, using both ARMA and GARCH, indicates strong similarities. This suggests that both models fit the data very accurately, and that they do provide a partial explanation of the pattern of equity returns on the Irish equity market.

Further research is clearly necessary, however. The study period for this paper coincides with the 1987 crash, an event which probably had a significant impact on levels of volatility on the market. Further research, comparing the relationships between volatility and return, both before and after that date, would confirm whether this is the case. Also, outside factors, including the performance of nearby major capital markets, may have an influence on levels of volatility on the Dublin market. This is an issue which deserves further investigation.

REFERENCES

Baillie, R.T. and DeGennaro, R.P. (1990). 'Stock Returns and Volatility', *Journal of Financial and Quantitative Analysis,* 25, pp.203-215.

Bollerslev, T. (1986). 'Generalised Autoregressive Conditional Heteroskedasticity', *Journal of Econometrics*, 31, pp.307-328.

Bollerslev, T., Engle, R.F. & Wooldridge, J.M. (1988). 'A Capital Asset Pricing Model with Time Varying Co-Variances', *Journal of Political Economy*, 96, pp.116-131.

Engle, R.F. (1982). 'Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation', *Econometrica*, 50, pp.987-1007.

Engle, R.F., Lilien, R.M. & Robins, R.P. (1987). 'Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model', *Econometrica*, 55, pp.391-407.

Fama, E.F. and French, K.R. (1992). 'The Cross-Section of Expected Stock Returns', *Journal of Finance*, 47(2), pp.427-465.

French, K.R., Schwert, G.W. & Stambaugh, R.F. (1987). 'Expected Stock Returns and Volatility', *Journal of Financial Economics*, 19, pp.3-29.

Hsieh, D.A. (1989). 'Modeling Heteroskedasticity in Daily Foreign-Exchange Rates', *Journal of Business and Economic Statistics*, 7, pp.307-317.

Merton, R.C. (1980). 'On Estimating the Expected Return on the Market: An Exploratory Investigation', *Journal of Financial Economics*, 8, pp.323-361.

Murray, L. (1992). 'A Test of the Stability of Beta Values Computed using Irish Data', Annual Proceedings, Irish Accounting and Finance Association, pp.105-128.

Poon, S.H. and Taylor, S.J. (1992). 'Stock Returns and Volatility: An Empirical Study of the UK Stock Market', *Journal of Banking and Finance*, 16, pp.37-59.

ACKNOWLEDGEMENTS

The authors would like to thank Tim O'Herlihy for his research assistance. They would also like to thank the anonymous referees for their helpful comments and suggestions. They also wish to acknowledge that any remaining errors are their own responsibility.