A CASE FOR CAPITAL ASSET CONSUMPTION ACCOUNTING:

THE DEMAND FOR LOCAL PUBLIC GOODS IN ONTARIO

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ABSTRACT

Public policies, including those with respect to make or buy decisions, estimates of the publicness and congestibility of local public goods, and of income and tax price elasticity are affected by the measure of expenditure of local governments. This paper presents the findings of an empirical study that supports the inclusion of asset consumption on the full accrual basis in the expenditure of local governments. The study estimates capital consumption using expenditure data for the period 1977 to 1988 for Ontario municipalities, a much longer time period than typically available for this type of study. The study contributes to the Accounting literature by providing theoretical and empirical support for accrual-based accounting for capital assets in a Not-for-Profit setting.

THE MODEL OF OUTPUT DEMAND

Research Question and Hypothesis

We are interested in the impact on the determinants and explanatory power of the demand model of local government expenditures when an estimate of the consumption of capital stock is included in local government expenditure.

We expect that there will be a significant improvement in the explanatory ability of the model using the proposed measure of capital consumption compared to the current measure of expenditures. This

improvement will be reflected in the conventional tests for significance and explanation of variance. The characteristics of the demand function, including publicness, and income and price elasticities, will be estimated. It is expected that these measures will be significantly different from that which earlier research has indicated, especially for services that are heavy users of capital. This is expected because previous measures of output mismeasured the level of expenditure.

The Conceptual Model of Local Government Output

A Samuelsonian public good is one whose benefits to each individual consuming it are not diminished by adding an additional consumer. In other words, the amount of the good consumed by one individual is equal to the aggregate supply. If public output of a pure public good is Q, this will create an aggregate benefit of Q x N where N is the population, because each individual receives a benefit of E from the total expenditure. Conversely, the aggregate benefit of output of a pure private good is just Q since each citizen's benefit is Q/N for the private good. This suggested the formulation $q = Q/N^\alpha$ for the individual benefit from consumption where the degree of publicness can vary between $\alpha = 0$ (pure public good) and $\alpha = 1$ (pure private good). These are polar cases; in practice it is expected that government-provided goods are partly congested so that α lies between zero and one.

The Median Voter Model

The objective of the median voter is to maximise utility (U):

$$U = u(Q/N^\alpha, X), \text{ or equivalently}$$

$$U = u(q, X)$$

where X is the composite other good and Q the public good where $Q = qN^\alpha$

The individual demand function for the public good is therefore of the form:

$$q = A(\tau P_Q N^\alpha)^\delta Y^e \text{ } [1]$$

Where A is a constant, $\tau P_Q N^\alpha$ is a measure of the tax price, and Y is the income of the median voter and δ and e are the price and income elasticties respectively. Since neither $\tau P_Q N^\alpha$ nor Y is observable directly,

researchers have developed various proxies for them. It is assumed that there is an efficient market for local public goods in a competitive market and that combined with the assumption of a homothetic production function for the services to be studied, allows expenditures to proxy for the level of output in dollars.[2]

$$So, q = A'\tau^{\delta}N^{\alpha\delta}Y^{e}$$

$$Q = qN^{\alpha} = E = A'\tau^{\delta}N^{\alpha(\delta+1)}Y^{e}, \text{ and}$$

in log form

$$lnE = c + \delta ln\tau + \alpha(\delta+1)lnN + elnY$$

The price elasticity is hypothesised to be negative and the income elasticity positive. The crowding parameter can be estimated from the coefficients on tax share and population as follows:

$$letting \ \phi = \alpha(1 + \delta)$$

$$then, \ \alpha = \phi/(1 + \delta)$$

Bergstrom and Goodman (BG, 1973) developed and tested a version of this model where the key independent variables were households rather than population and personal disposable income or average income rather than median. The tax share τ was assumed to be 1/HH. Bergstrom and Goodman assumed that prices of labour and capital were equal or at least in the same ratio across municipalities, allowing them to subsume P_Q, the price of the public good, in the constant. The BG (1973) model and several that followed it predict *total* expenditures. Because total expenditures are highly correlated with population, the resulting R^2s are high and misleading as an indicator of the demand determinants of the individual median voter. BG (1973) and later researchers have identified key socioeconomic variables as adding to the explanatory power of the median voter model. Population density, age of population and educational levels are typical.

Borcherding and Deacon (BD, 1972) developed a model of per capita expenditures on local public services and incorporated a wage rate differential in the tax price. Labour costs can be expected to differ because of local bargaining, local market conditions and other local competitive wages. In the BD (1973) paper and subsequent studies by other

researchers using a model of per capita expenditure, far lower R^2s and overall significance were observed as we would expect in a model of per capita expenditure.

Generally, the findings of the many researchers following BD (1972) and BG (1973) have been consistent with a conclusion that municipal services display the characteristics of private goods, namely, negative price elasticity, positive income elasticity and divisibility in supply. This does not necessarily mean that private goods are public goods — there are alternative explanations. An important one within the assumptions of the median voter model is that the economies of consumption (reflected in α) are offset by diseconomies of crowding. Put more succinctly, people will demand more public services as population density increases.

An alternative hypothesis for the provision of local public goods is the bureaucracy model. Niskanen (1971), Tullock (1967) and Gonzalez and Lemay (1985) argue that bureau heads, through information advantages and exploitation of voter transaction costs can manipulate the budget to achieve maximisation of discretionary budgets. Bureau heads equate marginal benefit and marginal cost but exact taxes at the average benefit level as would the monopolist in order to maximise discretionary expenditures. The bureaucracy model, through expenditure maximisation by the bureau heads, implies that changes in public expenditure will be more than proportional to changes in population, so ϕ in the model will be greater than one.

Gonzalez and Lemay's empirical results, as well as those of several other researchers, were consistent with the theory of monopoly provision of a public good with 'private good' characteristics. The bureaucracy model does not refute the median voter model, as the latter is a model of demand, while the bureaucracy assumption is one of supply. It does however, mean that the median voter model assumption of competitive supply is questioned. The bureaucracy model has serious implications for policy and for information disclosure as it suggests that the bureaucrats' information advantage in particular provides for the effective functioning of the monopoly behaviour. Residents are assumed to face the same tax prices across municipalities, so the price is subsumed in the constant.

METHODOLOGY

Proposed Demand Model

This paper proposes to test the demand for local public services using a variation of the Bergstrom and Goodman model and the proposed measure of expenditures including capital consumption.

Where quantity demanded is:

$$Q = qN^{\alpha} = E = A'\tau^{\delta}N^{\alpha(\delta+1)}Y^{e}, \text{ and}$$

$$\text{since } \tau = \sigma/N(\sigma \text{ is defined later})$$

$$E/N = A'\sigma^{\delta}N^{(\alpha-1)(\delta+1)}Y^{e},$$

where E/N is expenditures per capita, A' is a constant, δ is price elasticity of demand, $\tau = \sigma/N$ is the individuals' share of the tax bill, Y is their income and e is income elasticity of demand.

The demand for overall public expenditures per capita (E/N) then in log form becomes:

$$\ln E/N = c + \delta\ln\sigma + (\alpha-1)(\delta+1)\ln N + e\ln Y, \text{ or}$$

$$\ln E/N = c + \delta\ln\sigma + \phi\ln N + e\ln Y$$

The price elasticity is hypothesised to be negative and the income elasticity positive. The crowding parameter can be estimated from the coefficients on tax share and population as follows:

$$\text{letting } \phi = (\alpha-1)(\delta+1)$$

$$\text{Then } \alpha = \{(\phi)/(\delta+1)\}+1 \text{ the publicness parameter.}$$

As mentioned, socioeconomic variables are also important in the determination of per capita expenditures and tests are done for their significance. These will be included and their theoretical justification discussed, although it is not an area of primary interest. The primary interest of the paper is whether output is better measured with the proposed capital consumption measure.

Variables Definition

Dependent Variable: $E_t/N = (E_o + E_d)/N$

The proposal is to measure the output of the local government as the sum of current operating expenditures plus capital consumption. Current operating expenditures(E_o) are available for the three services to be tested and comprise wages and salaries, materials and maintenance costs and energy costs. Capital expenditure data are available by service and comprises expenditures to create or improve long-lived assets. Twelve years of data from 1977 to 1988 inclusive are used to construct capital expenditure and capital stock data for the individual years to be tested. The number of years available for tests depends on the asset life chosen. For example, asset lives of six years with the half year rule allow for tests to be run on the years 1983 to 1988. On the other hand, asset lives of 10 years limit tests to the years 1987 and 1988, since 11 years of data are required to develop the capital expenditure estimate. The capital expenditure data are adjusted for price changes to restate capital outlays in 1987 and 1988 dollars for the respective depreciation and capital stock figures for the two years, using the implicit price index for capital formation available from Statistics Canada. This is important for two reasons. First, it makes the capital expenditures comparable to operating expenditures by restating capital in 1987 and 1988 dollars. In addition, since expenditures proxy for the level of quantity demanded in this model, all expenditure information must be in the same dollars.

Independent Variables

τ: The specification of the tax price is an area of debate in the literature. The median voter model makes the assumption that the tax price is perceived to be a fixed proportion of the resident's share of total taxes paid. It is assumed that residents know their municipal tax bill and believe that their bill will change in proportion to municipal expenditures. They will know the cost to them of a given percentage change in municipal expenditures.

Several measures of the tax price have been used. In some cases the tax data were directly available, in others the tax bill was estimated from data on mill rates and residential and business property assessment. The simplest assumption is that the resident pays $1/N$ of the tax bill for the

local government. A more correct measure would be τ = median voter's tax bill/total tax bill for the community, as the tax share variable.

A significant proportion of municipal revenues is derived from non-residential tax revenue and from grants both conditional and un-conditional. It is reasonable to hypothesise that voters include con-sideration of the other revenue sources in their demand for local public goods. They may perceive the other sources as partly 'free' for example, in which case the assumption of tax price proportional to the share of total taxes is not appropriate. This can be a valid assumption because of 'fiscal illusion' or because tax exporting occurs. Fiscal illusion occurs when the voter perceives that the marginal burden of expenditure falls on others outside the local area. Tax exporting occurs when the burden really does fall on voters outside the municipality and the individual behaves rationally. There is theoretical and empirical evidence to support the tax exporting hypothesis although little for fiscal illusion. Essentially this hypothesis is that the burden (or part of) of the non-residential tax is borne by the consumers from other jurisdictions who purchase the goods and services produced in the municipality. As well, since provincial grants to a municipality are usually financed through general revenues, and are often used to help poorer communities to offer levels of service similar to richer ones, residents of the receiving communities, particularly a small one, are likely to consider these grants as a net transfer. Much of this discussion hinges on whether the local costs are perceived to be reduced at the margin by tax exporting.

The following tax share variable is typical of those used in the litera-ture to capture median voters' perceptions of their share of the marginal cost of the local public good. It attempts to capture the median voter's tax burden via the share of total revenues contributed by individual taxpayers. It is:

$$\tau = \sigma_1/N = (TXRF/TOTR)/N, \text{ where}$$

TXRF: Taxes: Residential and Farm for the local community

TOTR: Total revenues for the local community.

The ratio 'σ' is hypothesised to capture the ability of the local govern-ment to export taxes via the sales tax or as well to proxy for fiscal illusion.

N: Population of the local community or alternatively the number of households. Arguments will be presented in the paper for the alternative measures of N.

Y: Average income of residents of the local community as a proxy for the median income in the local community (see sample observation in the endnote[3]).

Tests of Models

Multiple regression is used to estimate the demand characteristics for Local Government services in the areas of, 'Transportation', 'Environment' and 'Culture and Recreation', for the years 1987 and 1988. These services are proposed because there are significant differences in the capital component of their expenditure and there is no evidence to reject the assumption of constant returns to scale in their production.[4] Multiple Regressions are run with and without the inclusion of capital consumption, to test for the sensitivity of results to the using up of the capital asset.

A typical problem with cross-sectional study and large differences in sample observations is heteroscedasticity. This should not be a significant problem with per capita expenditure data compared to some of the previous research using total expenditures. In addition, the log-linear approach should resolve problems of non-linearity. Scatterplots of the per capita expenditures are included in Appendix A, and suggest that this assumption is valid.

To recap:

The parameters will be estimated for the determinants of the expenditure level: $E_t/N = (E_o + E_c)/N$, where

E_o = operating expenditures

$E_d = D_t = C_t(d/2) + p_{t-1}C_{t-1}d + p_{t-2}C_{t-2}d....p_{t-1}C_{t-1}(d/2)$, where asset lives will differ depending on the arguments made for accelerated depreciation and asset life.

The estimating equation is:

(1) $\ln(E_t/N) = c + \delta\ln\sigma + (\alpha-1)(\delta+1)\ln N + e\ln Y$

Where $\phi = (\alpha-1)(\delta+1)$ and $\alpha = \phi/(\delta+1) + 1$ as before.

This is referred to as Model$_D$

The results will be compared to the equations estimated using two other measures of expenditures.

The first is the traditional equation from the literature which is simply:

(2,i) $\ln E_o/N = c + \delta\ln\sigma + (\alpha-1)(\delta+1)\ln N + e\ln Y$

Where E_o includes *operating expenditures* only in the estimate for the dependent variable. This is referred to as Model$_O$.

The second equation reflects the way expenditures are currently *reported* by local governments.

(2,ii) $\ln(E_t/N) = c + \delta\ln\sigma + (\alpha-1)(\delta+1)\ln N + e\ln Y$

Where $E_t = E_o + E_c$, where E_c is current capital outlays. This is referred to as Model$_C$.

EMPIRICAL RESULTS

Independent Towns and Cities

Tables 1 and **2** summarise the results for the three models tested and presents F-tests for significant differences in variance in **Table 1**, while elasticities and the publicness measure are presented in **Table 2**.

F-Tests

The F-tests in **Table 1**, column two, indicate that there is little difference in the explanation of variance in the proposed Model$_D$ compared to the measure of current expenditure only, Model$_O$ except for Environment Services where the improved explanatory power with the proposed measure of expenditure is significant at the 5 per cent level.

The F-tests in **Table 1**, column three, however, indicate that there is a significant improvement in the explanation of variance in the proposed $Model_D$ compared to the current method of reporting of capital in $Model_C$. This is true for All Services combined and for each of the tested services. The individual services are all significant at the 1 per cent level. It should be repeated that $Model_C$ represents the way municipalities report, while $Model_O$ uses the measure of expenditure that is typically used in the research on expenditures of local governments.

Elasticities and Publicness

Table 2 summarises the results of the tests for elasticities in the three models for All Services and for the three individual services tested. All Services means all services offered by the local government, not the sum of the three in the table.

All Services

The 'All Services' column indicates significance on all the independent variables in the proposed $Model_D$. The signs on the coefficients are also as hypothesised. The results suggest that as incomes rise, per capita expenditures on local public goods increase although less than proportionally. There is a strong negative relationship between the tax share borne by the local resident and per capita expenditures, indicating that consumers are sensitive to the ability of the local government to export some of the tax burden or to attract funding that is not paid by the local resident. The elasticity of expenditures with respect to population size is significant, but small. This suggests that, as population increases, there is little jointness in consumption or economies of scale as we might expect with a public good, or alternatively the jointness in consumption is offset by congestion in the provision of the local good or service. The degree of publicness for All Services is 1.21, indicating that, on average, local public goods display a high degree of privateness in consumption or congestion in provision. This likely reflects the high proportion of transfer payments included in the total local expenditure figure. It is likely of course that different services display different characteristics. A discussion of some of those characteristics follows.

Similar results are achieved for $Model_C$ with respect to significance. Note that the elasticity with respect to income, however, is much lower in the proposed $Model_D$ compared to $Model_C$. This is important as it suggests that local public expenditure is less responsive to changes in income levels than indicated by the current level of reporting for the years tested.

More importantly for the hypothesis of this paper, income level drops out as a significant variable in $Model_O$. It is not plausible theoretically, or based on other research on demand models, that expenditure levels per capita are not in part determined by income levels. This is an important result as $Model_O$ is the typical model used in the research, and these results would suggest that the model of demand is improved by the inclusion of a measure of capital consumption.

Transportation Services

The results for Transportation Services are equally interesting. $Model_D$ shows significance and the correct sign on the income and share variables while tax share is not significant in $Model_C$, while income is not

Table 1: F-Tests
Demand for Local Public Goods in Ontario
Independent Towns and Cities

Service↓ Expenditure measure→	Operating expenditures only $Model_O$: $Model_D$	Expenditures including current capital $Model_C$: $Model_D$
All Services	1.12	1.40**
Transportation	1.02	1.69***
Environment	1.40**	2.20***
Culture and Recreation	.86	1.72***

F-test: compares the residual errors (SSEs) of the models with the alternate measures of expenditures to the proposed measure (i.e. with depreciation)

Significant at: 10%*, 5%**, 1%***

Table 2: Elasticities and Adjusted R^2
Demand for Local Public Goods in Ontario
Independent Towns and Cities, 1987/88

Model$_D$: Expenditures include depreciation

Test↓ Service→	All Services	Trans- portation	Environ- ment	Culture & Recreation
Adj.R^2	.64	.24	.47	.34
Elasticities				
Dummy for year	.12***	.13**	.15**	.10*
Income (e)	.49***	.90***	.63**	1.6***
Tax share (δ)	-.72***	-.37***	-.67***	-.20**
Population (φ)	.06***	.01	.06**	-.12***
Publicness (α = φ/(δ+1) +1)	1.21	1.02	1.18	.75
F-test	53.6	10.1	26.7	16.2

Model$_C$: Expenditures incl. *Current Capital*

Test↓ Service→	All Services	Trans- portation	Environ- ment	Culture & Recreation
Adj.R^2	.54	.13	.24	.18
Elasticities				
Dummy for year	.14***	.18**	.13	.09
Income (e)	.60***	1.09***	.67*	1.4***
Tax share (δ)	-.65***	-.19	-.49***	-.20
Population (φ)	.05**	-.03	.08**	-.10***
F-Test	35.6	5.2	10.4	7.6

Model$_O$: Expenditures: *Operating only*

Test↓ Service→	All Services	Trans- portation	Environ- ment	Culture & Recreation
Adj.R^2	.60	.14	.40	.29
Elasticities				
Dummy for year	.13***	.14**	.15**	.11**
Income (e)	.26	.44	.75**	1.4***
Tax share (δ)	-.70***	-.26**	-.50***	-.12
Population (φ)	.09***	.04	.08**	-.08***
F-Test	45.1	5.8	18.3	13.1

Significant at: 10%*, 5%**, 1%***

significant in $Model_O$. The elasticity of transportation expenditures with respect to income is .9, or almost proportional, indicating a fairly elastic demand for this service. The negative elasticity with respect to tax share is of the correct sign and the relatively low elasticity indicates that per capita expenditure drops slightly as the tax share increases. The publicness parameter of ≈ 1.00 suggests that there is little jointness in the consumption of transportation services.

Environment Services

All the independent variables are significant and the coefficients have the expected sign in all three models, except for the dummy variable in $Model_C$. This would suggest that the expenditures on Environment services are fairly stable from year to year, so the 'steady state' achieved by depreciation is fairly similar to the annual outlay on capital expenditures.

Culture and Recreation

The results for the Culture and Recreation service are particularly interesting since we might expect by hypothesis that this service is the most 'public' of the three presented here.

The significance and signs on all three key variables are as hypothesised for $Model_D$, while the tax share drops out as significant in the other two models. Notice the high elasticity with respect to income for the Culture and Recreation service in $Model_D$, suggesting that demand for this service increases more than proportionally with income, an appealing result intuitively. The publicness parameter is .75, indicating significant jointness in consumption in the cultural and recreation service, also an appealing intuitive result.

EMPIRICAL RESULTS

Lower-Tier Towns and Cities

F-tests

The F-test results are almost identical to the results for the Independent Towns and Cities. The proposed $Model_D$ is clearly superior in explaining variance in local expenditures compared to the current method of

reporting reflected in $Model_C$ for All Services and for the individual services. Compared to $Model_O$, the proposed model improves the explanation of variance for Culture and Recreation services.

Elasticities

All Services: All three models show significance on the three key variables and the signs are in the expected direction. Note, however, that there are significant differences in the elasticity on income between the three models, varying from a low of .65 for $Model_O$ to a high of 1.01 for $Model_C$. If we accept $Model_D$ on theoretical grounds, it is apparent that expenditures on the capital component are more responsive to changes in income than operating expenditures, an important result.

Transportation: Only the tax share variable is significant and is of the correct sign. This may reflect provincial initiatives to harmonise roads across the province, resulting in similar per capita expenditures that are relatively unaffected by the income levels in the community. In addition,

Table 3: F-tests		
Demand for Local Public Goods in Ontario		
Lower Tier Towns and Cities, 1987/88		
Service↓ Expenditure measure→	Operating expenditures only $Model_O : Model_D$	Expenditures including current capital $Model_C : Model_D$
All Services	1.00	1.63[***]
Transportation	1.08	1.36[**]
Environment	1.09	1.20[*]
Culture and Recreation	1.20[*]	1.53[***]

F-test: compares the residual errors (SSEs) of the models with the alternate measures of expenditures to the proposed measure (i.e. with depreciation)

Significant at: 10%[*], 5%[**], 1%[***]

Table 4: Elasticities and Adjusted R^2
Demand for Local Public Goods in Ontario
Lower-Tier Towns and Cities, 1987/88

Model$_D$: **Expenditures include depreciation**

Test↓ Service→	All Services	Trans- portation	Environ- ment	Culture & Recreation
Adj.R^2	.60	.15	.10	.27
Elasticities				
Dummy for year	.08**	.05	.08	.10*
Income (e)	.81***	00	2.17***	.89***
Tax share (Λ)	-.79***.	-.43***	-1.25***	-.68***
Population (φ)	.02	.01	00	00
Publicness (I = φ/(Λ+1) + 1	1.09	1.02	1.00	1.00
F-Test	45.0	6.3	4.5	11.0

Model$_C$ **Expenditures incl.** *Current Capital*

Test↓ Service→	All Services	Trans- portation	Environ- ment	Culture & Recreation
Adj.R^2	.49	.04	.10	.27
Elasticities				
Dummy for year	.08**	.04	.13	.09
Income	1.01***	.09	2.3**	1.41***
Tax share	-.77***	-.31**	-1.13***	-.78***
Population	.03	00	.04	00
F-Test	29.1	2.2	4.2	11.9

Model$_O$ **Expenditures:** *Operating only*

Test↓ Service→	All Services	Trans- portation	Environ- ment	Culture & Recreation
Adj.R^2	.61	.16	.06	.35
Elasticities				
Dummy for year	.08***	.05	.12	.11**
Income	.65**	-.27	1.6*	.78***
Tax share	-.78***	-.36**	-.97**	-.74***
Population	.04**	.00	00	.01
F-Test	47	6.5	2.9	16.9

Significant at: 10%*, 5%**, 1%***

because all the towns and cities are part of regional structures in relatively well-developed areas, there may be more effort within the regions to have similar transportation facilities as we see in the independent towns and cities.

Environment: Environment services are highly sensitive to changes in income, indicating that expenditures increase by double the relative change in income. The demand for these services is also highly sensitive to the tax share. These results are appealing intuitively and are similar to the independent towns and cities.

Culture and Recreation: The sign and direction on the income and share variables is significant and the correct sign. The responsiveness of demand is not as elastic as the independent towns and cities. This result could be consistent with the zoo effect. Since most of the cities in the regional structure are large, they have typically reached the size at which facilities are replicated with each population block.

CONCLUSIONS

This paper proposes that the measure of annual expenditures of local governments should include an estimate of the consumption of capital assets. It is argued that the demand for local government services is better measured when capital consumption is included in the measure of expenditures, and that the accounting concept of depreciation is proposed as a reasonable proxy for this measure. Depreciation is estimated for the years 1987 and 1988 for the 62 independent towns and cities and the 60 cities that are organised in the regional structure, and is added to the annual operating expenditures for the above cities. The model of demand including depreciation is compared to the traditional model in the literature and to the way expenditures are currently reported by the municipalities. The results suggest that variance in per capita expenditure is better explained using depreciation for all services tested as opposed to the way expenditures are currently reported. The variance for the Environment Services is better explained by the proposed model as opposed to the traditional measure in the literature.

The results indicate that income and tax share are significant and of the correct sign in the demand for all three services tested when we include

depreciation in the measure of expenditures. This result is not found with the other two measures of expenditure. In addition, the magnitude of the coefficients differs in the models tested, suggesting that if we accept the proposed model on theoretical grounds, it has important research and policy implications. Appealing intuitive results are evident for the Culture and Recreation service where the elasticity of demand with respect to income is high and a significant degree of jointness in consumption is suggested for this service. Jointness may be more associated with the capital component than with operating expenditures.

NOTES

[1]   where q = quantity demanded = $AP^{\delta}Y^{e}$

It is shown below that δ is the price elasticity and e the income elasticity, proving indirectly that $AP^{\delta}Y^{e}$ represents the quantity demanded where we assume that price and income elasticities are constant and the relationship is log-linear.

Price elasticity = $dq/dp*P/q$, by definition.

and $dq/dp = AY^{e}\delta P^{\delta-1}$

So, $dq/dp*P/q = (AY^{e}\delta P^{\delta-1})*P/(AP^{\delta}Y^{e}) = \delta$, as all other terms cancel.

Q.E.D.

The proof that income elasticity = e is identical.

[2]   The Klein-Rubin or Stone-Geary utility function shows that the optimal quantities give the demand function:

$q_1 = g_1 + \beta_1/p_1(Y - p_1*g_1 - p_2*g_2)$

where q = quantity demanded, p = prices and g = subsistence quantities, Y = income.

Multiplying by p_1 gives

$p_1q_1 = p_1g_1 + \beta_1 1(Y - p_1*g_1 - p_2*g_2)$,

which is linear in prices and income and allows the expenditure level, p_1q_1 to proxy for the level of demand.

[3] Sample observation City of Ottawa 1987 Total expenditures

Dependent Variable:	
Expenditure per capita E/N	$793
Independent Variables	
Tax share 1: σ_1	.88
Tax share 2: σ_2	.712
Population N	313,015
Income per household Y	$52,921

Explanations of data points:

Sample Observation * denotes a variable needed for equations (1) or (2).

City of Ottawa: 1987 $000 (except where noted)

Expenditures

E_o: Operating expenditures			209,243
E_c: Capital expenditures:	1987:	42,061*10%	4206
	1986:	41,675*20%	8335
	1985:	38,795*20%	7759
	1984:	31,526*20%	6305
	1983:	42241*20%	8448
	1982:	43,250*10%	4325
	Total capital		39,378

$E = E_o + E_c$ = Total expenditures $\qquad$ $248,261

E/N = $248,261,000͵313,015 $\qquad$ $793

Tax share variables

*σ_1 = (TXRF/TOTR) = .88, where

TXRF: Taxes: Residential and Farm $\qquad$ 101,682

TOTR: Total revenues ($234,052 + $31,986, operating plus capital) $\qquad$ 266,038

* σ_2 =EARF/EATO) = $\qquad$ .712, where

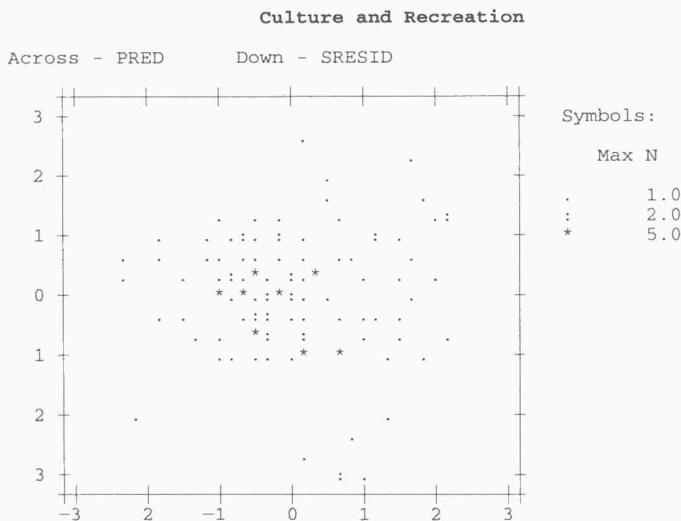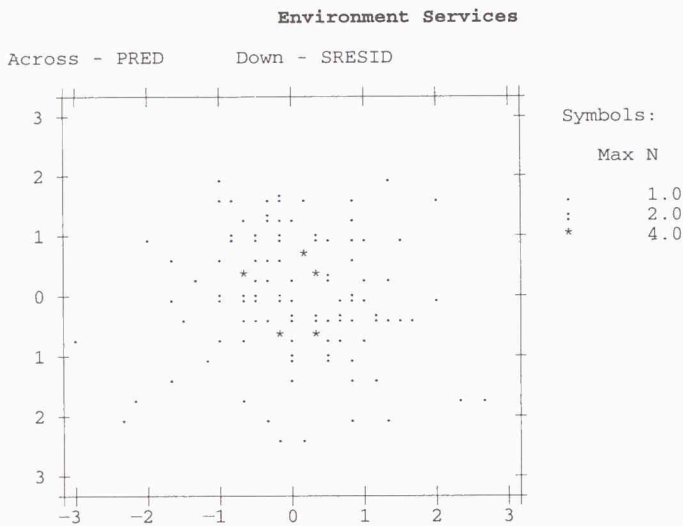|	
EARF = Equalised Assessment, Residential and Farm	20,180,000
EATO = Equalised Assessment, Total	28,303,000
Income Measure	
* Y: Income per Household	52,921
Population	
* N: Population	313,015

[4] Oates (1987) has proposed that economies of scale with respect to population size seem to be contained within population 'blocks' of 10,000 or so. As a result, as population size increases, services are replicated and economies of scale are subsumed within the data. As these samples of cities range from about 10,000 to 500,000 we would not expect to find economies of scale. MacMillan, Wilson and Arthur (1981) segregated a sample of Ontario towns into those less than 10,000 and those more than 10,000 and found results consistent with this thesis.

In addition, the author is currently undertaking research that addresses the question of whether economies of scale are evident with respect to the level of output of municipal services. This study is limited to the level of expenditures of local government with respect to population as the scale measure.
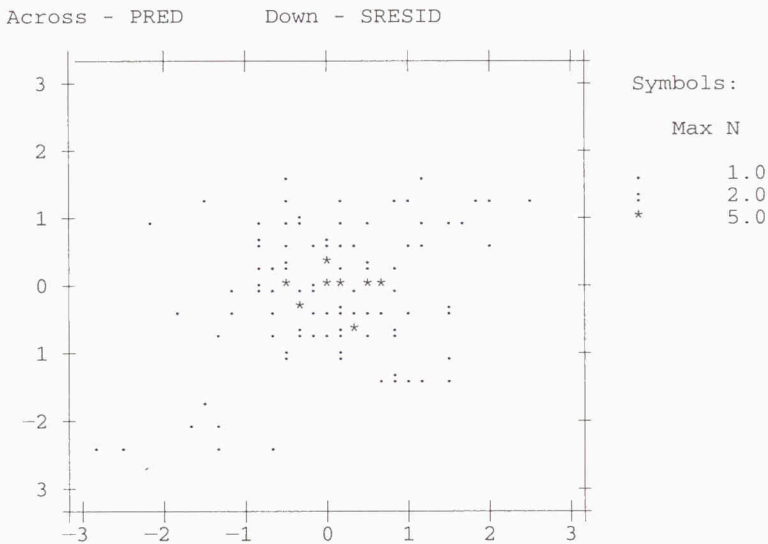
Ron Eden

APPENDIX A

Standardised Scatterplots — Independent Towns and Cities

PRED is the standardised predicted value of expenditure per capita.
SRESID is the standardised residual.

Environment Services

Across - PRED   Down - SRESID

Culture and Recreation

Across - PRED   Down - SRESID

Transportation Services

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Across  -  PRED        Down  -  SRESID
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