INVESTMENT HORIZONS, TIME DIVERSIFICATION AND SUSTAINABLE WITHDRAWAL RATES FOR A RETIREMENT INVESTMENT IN UK MARKETS

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ABSTRACT

We examine the sustainability of wealth and associated risks faced by a retiree in the drawdown phase of retirement. Risks are assessed using a range of commonly recommended withdrawal rates, and applied to common investment strategies in the UK markets. We consider the issue of time diversification of risk, in the context of expected retirement horizons. Our results demonstrate that after allowing for compounding effects and for regular investment withdrawals, the risk of financial ruin increases with the length of the retirement horizon, contrary to the premise of time diversification of risk.

INTRODUCTION

Market fluctuations and regulatory changes regarding pension investments have, in recent years, impacted on the potential experience of pensioners who expect to require a regular income. The issue of sustainable spending rates for retirees has taken on an added importance since the global financial crisis, due to the substantial losses that pension funds suffered during 2007–2008, and the mixed performance of markets and funds in the post-crash period. The importance of sustainable spending rates is also underscored by increasing longevity. For example, average life expectancy at birth in the United Kingdom (UK) continues to increase, reaching

79.5 years for males and 83.2 for females in 2012–2014.[1] Life expectancy at age 65 has also reached 18.8 years for males and 21.2 years for females. Given the importance that retirement asset pools will play in the life of those who will live longer, and the possibility that the savings pool available to a retiree at the point of retirement may be lower than anticipated due to poor market conditions, it is important that the manner in which savings pools are utilised is carefully examined. Commentators charge that too little concern is paid by the financial planning industry to the concept of sustainable spending. For instance, Arnott (2004, p. 6) states 'Our industry pays scant attention to the concept of sustainable spending, which is key to effective strategic planning for corporate pensions, public pensions, foundations, and endowments – even for individuals.'

In our paper, we consider the situation of a new retiree who at the time of retirement needs to plan for monthly withdrawals from their savings pot for living expenses, while ensuring that portfolio value is maintained at a level sufficient to support the withdrawal stream for the rest of their life. If the withdrawal rate is too high the retiree faces the prospect of running down the savings pool prematurely, resulting in financial ruin. If the withdrawal rate is too low, the retiree is deprived of a better standard of living.

If future investment returns are known with certainty, the solution to the determination of sustainable withdrawal rates is a present value computation, dependent on assumptions regarding lifespan following retirement. But there is uncertainty in expected investment returns and an unknown lifespan, so it is necessary to balance the amount of withdrawal with the risk of exhausting available capital. There are other contexts where the same computational principles apply. For example, the trustees of an endowment such as a scholarship fund need to determine the maximum sustainable scholarships that can be offered while maintaining desired safety levels in portfolio value. Financial institutions such as insurance companies also face a similar question in pricing annuity products.

Making longer-term projections of global economic conditions or of market performance is a hazardous guess at best, given the prevailing economic uncertainties. In an environment where no one can make clear predictions about future market performance, perhaps a sensible approach to take in retirement financial planning is to base future market expectations on the past experiences and performance of markets, with some added sensitivity analyses for estimating risk. The financial experiment conducted in this paper follows this approach, where future projections are made by replicating past market performances in a bootstrap simulation framework, for the purpose of formulating sustainable withdrawal rates and estimating risks associated with those outcomes, as measured by probabilities of financial ruin and of wealth shortfall.

We also specifically address the issue of time diversification of risk in relation to retirement planning horizons. The conventional wisdom of time diversification would suggest that a retiree with a long investment horizon would have lower investment risk, but at shorter horizons, risk levels would increase. We examine this issue in the context of the retiree who makes monthly withdrawals for consumption over their retirement life, and who views risk in terms of two alternative measures: the probability of facing financial ruin and the probability of facing a shortfall in wealth. We examine the variation of these risks as the investment horizon shortens.

Risk outcomes are examined for a range of hypothetical investment strategies, consisting of investments in pure UK asset classes such as all equities (ALL), all bonds (BOND), an equally weighted equity/bond portfolio (SHBD), and a glide path of an equally weighted equity/bond portfolio increasing in its bond exposure as the retirement horizon decreases (GLIDE). Simulations are carried out for a range of withdrawal rates commonly adopted in the finance industry. We concentrate on the UK market, as it offers a full range of suitable investments.

In the next section, we review relevant literature and the methodologies we use in the paper. The sample data and main results are described in further sections, and a final section summarises and concludes.

LITERATURE AND METHODOLOGY

Researchers in the financial planning industry have been using data-centric approaches to estimate a maximum sustainable withdrawal rate that will allow a retirement asset pool to fund a full retirement period. An early study that has significantly influenced industry practice is Bengen (1994). In this study, 65 years of United States (US) equity market data between 1926 and 1991 were analysed to search for the highest withdrawal rate that would sustain an equally weighted portfolio of stocks and bonds over a 30-year retirement horizon. Based on his results, Bengen (1994) prescribes a 4 per cent withdrawal rate. This rule has come to be known as the '4 per cent rule' and investment advisors in the US, the UK, and other developed markets widely follow the policy of recommending a withdrawal rate of 4–6 per cent for retirement portfolios. The spending rate is increased each year by the rate of inflation in order to maintain a constant level of consumption over a retirement horizon, usually assumed at thirty years.

Other studies examine variations on this basic theme. Pye (2000) utilised a bootstrap simulation because it provides a framework for mimicking investors who make sustainable withdrawals. Investment choices can be evaluated over time, without needing to make any further assumptions about risk preferences. Hickman et al. (2001) examine the interplay between horizon and risk/return performance for various asset classes. Using analytical techniques, Milevsky and Robinson (2005) identify the withdrawal rate, the asset allocation decision, and investor mortality, as impacting on the likelihood of financial ruin. Basu, Byrne and Drew (2011) examine the accumulation phase of retirement planning, and the benefits or otherwise of asset allocation changes from a pure glide path that is commonly adopted by lifecycle or target-date retirement funds. The focus of many of these studies has been on investors and markets based in the US. It is surprising that the issue of sustainability in retirement planning in the UK and Ireland has received relatively limited attention.

The issue of time diversification is also relevant. Commonly cited in practitioner literature, the notion is that periods of above-average returns will offset periods of below-average returns, over a sufficiently long time horizon (Kritzman, 1994).

Assuming that asset returns are independent from one year to the next, the standard deviation of annualised return will diminish with time. The distribution of annualised returns will consequently converge as the investment horizon increases. A criticism of the analogy is that successive annual returns will be compounded into a total multi-year return, rather than being simply averaged. The risk should therefore be greater the longer an asset is held. This increased risk will, however, be offset by higher returns over a longer horizon, so it is unclear as to whether an extended investment horizon will be beneficial to investors. Samuelson (1969) formalises this point, as he demonstrates that although the annualised dispersion of returns converges towards expected return as the time horizon extends, the dispersion of terminal wealth also diverges from expected values. This result implies that although an investor is less likely to lose money over a long horizon than over a short horizon, the magnitude of potential loss increases with the duration of the investment horizon. Our paper offers a further contribution to this debate, as we consider this issue within the context of investment funds that are subject to regular withdrawals. We will examine this issue by considering both the risks associated with a diminishing retirement horizon, and the risks associated with an increased amount of periodic withdrawals. We apply a bootstrap simulation approach to investigate risk exposures and wealth-loss probabilities in retirement portfolios. We describe our bootstrap simulation approach in the following sub-section.

Bootstrap Methodology

The bootstrap re-sampling technique is widely used in finance to expand data size when the number of observations is relatively small. The span of available UK historical equity index return data and bond return data is quite limited. When historical experience needs to be replicated into the future, the available dataset can be extended by randomly re-sampling the data with replacement. The regenerated dataset can provide a sufficient number of independent samples needed to generate long series of return paths and a distribution of end-of-period wealth outcomes.[2] Because re-sampling is done with replacement, a particular data point from the original dataset can appear multiple times in a given bootstrap sample. The probability distribution of future outcomes must represent past data. To compute bootstrap samples, we start with the set of monthly return observations in the data series and repeatedly draw random observations with replacement until the required number of observations is drawn for a single complete run of wealth path computations. This results in one bootstrap sample. By repeating this process 500 times, we obtain 500 sample runs with randomly selected starting observations for the investment returns. The computational results from the 500 samples provide a frequency distribution of values from which probabilities of outcomes can be computed.

For illustrative purposes, we assume a retiree is 60 years of age on retirement, with a savings pool of £100, and plans for a retirement period of up to 30 years, potentially taking them to a maximum lifespan of 90 years. Over a 30-year period the retiree will make 360 monthly withdrawals. For shorter horizons of 10 and 20 years, we allow for 120 and 240 monthly withdrawals respectively. Wealth paths are simulated for withdrawal rates of 4 per cent, 5 per cent and 6 per cent per year on the initial wealth. Investment returns are continuously compounded, and they are stated in nominal terms, which would incorporate inflation effects. To convert the withdrawals also to nominal terms, we raise the monthly withdrawal rate by the monthly inflation rate, based on an annual inflation rate of 3 per cent. We believe that this is a reasonable estimate for the future UK inflation rate.

The model is calibrated as follows. Investment portfolio values at the beginning and the end of month *t* respectively are V_t and V_{t+1}. Portfolio return for the month is R_t, consisting of the capital gains and other cash returns such as dividends, assuming monthly compounding. The withdrawal, W_t, is assumed to be made at the end of each month. Value of the investment portfolio at the end of month *t* therefore depends on:

$$V_{t+1} = V_t(1+R_t)-W_t \quad (1)$$

The value of an investment portfolio consisting of shares and bonds (SHBD) invested in the proportion λ in equity and 1–λ in debt is:

$$V_{t+1} = V_t[1+\lambda R_{St}+(1-\lambda)R_{Bt}]-W_t \quad (2)$$

The withdrawal amount is increased at the rate of monthly inflation 0.0025, so as to maintain a constant level of real consumption. R_{St} and R_{Bt} represent monthly return on the equity (ALL) and bond (BOND) investments respectively.

Our investigation of risk exposures and probabilities of financial ruin is based on the distributional return and risk properties of UK equities and bonds. We maintain the same assumptions of an initial savings pool of £100 and retirement periods of ten, twenty and thirty years. Results are computed for the ALL, BOND, SHBD and GLIDE strategies. We report mean and standard deviation of end-of-period wealth values and minimum values. We also compute probabilities of financial shortfall (i.e. wealth falling below the initial investment value of £100) and of financial ruin (i.e. wealth falling below zero, or fund exhaustion). The coefficient of variation in end-of-period wealth values is also reported, as it offers an indication of risk/return relationships.

DATA

In our study, we use index measures covering the equity market and the government debt market. Other major investment classes in the UK have not been included, as regular monthly indices of performance are not available over a sufficiently long historical time period. The FTSE All Share Index is selected to represent the equity market, as it is the longest available data series. It covers approximately 98 per cent of total market capitalisation.[3] When estimating returns on the FTSE All Share Index, we use total return values, as they include an adjustment for reinvested dividends. Twenty-year-long bond yield data provide the base from which we estimate return on bonds.[4] In order to compute returns, we estimate the bond price at the start of each month, using yield curve data. Allowing the bond term to reduce by one month, we estimate an end-of-month value, using the yield on that date. Return is then calculated as the change in bond value over the month, plus accrued interest, divided by the start-of-month value. Every measure of monthly bond return comes from repeating this procedure. To facilitate these calculations, an assumption regarding the level of bond coupon payment is needed. A reasonable estimate is that each coupon payment is equal to the average of yield values over the previous three years.[5]

Monthly equity index values and long bond yields are both available from January 1965. We therefore compute monthly returns for each series from January 1965 to December 2014. This is a relatively long data series, covering a number of periods of both recession and growth, so we believe that it offers an appropriate dataset from which to analyse future sustainable withdrawal rates.

RESULTS

Summary Statistics

In Table 1, we report summary data for monthly return on the equity (ALL) and the bond (BOND) asset classes. We present the mean values, standard deviations, skewness and kurtosis. As expected, an investment in BOND provides lower average monthly returns and a lower standard deviation, indicating lower risk exposure. We also note considerably higher levels of positive skewness in BOND returns.

TABLE 1: SUMMARY RESULTS – SAMPLE DATA FROM JANUARY 1965 TO DECEMBER 2014 (NUMBER OF OBSERVATIONS = 598)

Return Series	Mean	Std. Dev.	Skewness	Kurtosis
ALL	0.0096	0.0543	0.1126	8.3748
BOND	0.0079	0.0269	0.5888	1.3151

Notation:
ALL FTSE All Share index monthly return
BOND Government Bond index monthly return

Bootstrap Results

In Tables 2 and 3, we identify apparent time diversification benefits. In Table 2, we report holding period returns and risks of each asset class. Returns are from re-sampling with the bootstrapping technique. In the re-sampling procedure, they are drawn independently to compute monthly return, and these returns are summed up to give the holding period return. We therefore do not allow for the impact of compounding on holding period return. This is repeated 500 times and the mean, standard deviation and coefficient of variation (CV)[6] of the 500 holding periods are reported. We show both total horizon returns and annualised returns.

We find that both total horizon return and standard deviation of return increase over longer holding periods. This is unsurprising. An examination of annualised values provides a more informative comparison. Annualised return increases slightly with increases in the holding period, but standard deviations decline monotonically

TABLE 2: RETURNS AND RISKS AT DIFFERENT HOLDING PERIODS (BASED ON 500 BOOTSTRAPPED SAMPLES)

		10 Years	20 Years	30 Years
Total horizon return				
ALL	Mean	1.190	2.334	3.990
	Std. dev.	0.591	0.748	0.726
	CV	0.497	0.320	0.182
BOND	Mean	0.980	1.898	3.235
	Std. dev.	0.354	0.456	0.443
	CV	0.361	0.240	0.137
Annualised return				
ALL	Mean	0.119	0.131	0.133
	Std. dev.	0.059	0.042	0.024
	CV	0.497	0.321	0.180
BOND	Mean	0.098	0.106	0.107
	Std. dev.	0.035	0.025	0.014
	CV	0.361	0.236	0.131

Notation:
ALL FTSE All Share index monthly return
BOND Government Bond index monthly return

as the holding period lengthens. Coefficients of variation also decline as holding periods extend. This is evident if either total horizon returns or annualised returns are considered. These are the apparent benefits of time diversification, previously reported in Alles and Murray (2009). They are evident for both asset classes. They are the result of the mitigating impact of above and below average monthly returns on each other, giving the appearance of reduced risk.

In Table 3, we explore the distribution of wealth outcomes from an investment in each asset class, across a range of holding periods. We assume an investment of £100 in each asset class and we compare the characteristics of the distribution of end-of-period wealth, and how the distribution of end-of-period wealth changes for holding periods of ten, twenty and thirty years. To compute end-of-period wealth from an investment of £100 in an asset class we sum monthly returns obtained with the re-sampling methodology for the number of periods required for each holding period and we add the original investment. It should again be noted that we do not allow for the impact of compounding when estimating end-of-period wealth. We confirm that equities offer greater wealth potential than bonds. Expected value following a ten-year investment in equities is £222, while bonds provide £201. We note a relatively small increase in standard deviations of end-of-period wealth values as holding periods extend from ten to twenty years, followed by a small reduction if holding periods extend further to thirty years. Coefficients of variation therefore decline, again indicating the apparent benefits of time diversification.

TABLE 3: STATISTICS OF END-OF-HOLDING-PERIOD WEALTH DISTRIBUTIONS

	10 Years	20 Years	30 Years
ALL			
Mean	222	335	504
Std. dev.	60	79	69
CV	0.27	0.24	0.14
Median	226	339	500
Minimum	96	120	343
01-%ile	102	169	367
10-%ile	140	225	413
90-%ile	302	434	597
99-%ile	343	486	707
BOND			
Mean	201	288	425
Std. dev.	36	47	45
CV	0.18	0.16	0.11
Median	208	296	431
Minimum	95	142	293
01-%ile	122	168	314
10-%ile	150	221	364
90-%ile	244	342	478
99-%ile	262	367	498

Notation:
ALL FTSE All Share index monthly return
BOND Government Bond index monthly return

End-of-Retirement-Horizon Wealth and Probabilities of Wealth Loss

In Table 4 we identify wealth outcomes after a ten-year, a twenty-year and a thirty-year horizon for the retired investor with an initial investment fund of £100, based on the monthly withdrawal and the reinvestment and compounding process shown in Equations (1) and (2). We assess the impact of annual withdrawal rates of 3 per cent, 4 per cent, 5 per cent and 6 per cent at monthly intervals. For purposes of comparison we also examine the impact of no withdrawals, as this will correspond to the holding period outcomes in Table 3. We report the mean and standard deviation of end-of-horizon wealth, the coefficient of variation, minimum end-of-period wealth, the probabilities of financial ruin and the probabilities of financial shortfall for the all equity (ALL), all bond (BOND), balanced equity/bond (SHBD), and glide path (GLIDE) strategies.

For all investment strategies, regardless of horizon, mean end-of-horizon wealth typically declines as the withdrawal rate is increased. If we consider a twenty-year investment in ALL, mean end-of-horizon wealth reduces from £756 to £584 as annual withdrawal rates increase from 3 per cent to 6 per cent. Minimum values also decline from -£19 to -£399. The probability of financial shortfall (PS) increases

TABLE 4: STATISTICS ON END-OF-RETIREMENT-HORIZON WEALTH DISTRIBUTION AND PROBABILITIES OF WEALTH LOSS

	10-Year Horizon						20-Year Horizon						30-Year Horizon					
W/D	Mean	SD	CV	Min	PS	PR	Mean	SD	CV	Min	PS	PR	Mean	SD	CV	Min	PS	PR
Panel A: ALL																		
0%	308	211	0.69	26	0.06	0	954	975	1.02	44	0.01	0	3165	3988	1.26	13	0	0
3%	266	180	0.67	21	0.11	0	756	821	1.08	-19	0.06	0	2249	2745	1.22	-55	0.04	0.01
4%	232	187	0.80	7	0.22	0	764	879	1.15	-1	0.11	0	2181	3809	1.74	-376	0.09	0.04
5%	216	161	0.74	4	0.22	0	585	754	1.28	-77	0.17	0.04	1852	3796	2.04	-990	0.17	0.08
6%	203	165	0.81	-1	0.29	0	584	1054	1.80	-399	0.25	0.09	1498	2713	1.81	-727	0.21	0.12
Panel B: BOND																		
0%	146	82	0.12	103	0	0	219	39	0.17	120	0	0	326	75	0.22	177	0	0
3%	110	16	0.15	62	0.27	0	127	31	0.24	53	0.18	0	156	49	0.31	46	0.11	0
4%	100	16	0.15	63	0.52	0	97	27	0.27	35	0.58	0	98	44	0.45	8.4	0.55	0
5%	88	14	0.16	53	0.81	0	67	25	0.38	10	0.89	0	42	37	0.88	-58	0.93	0.12
6%	75	14	0.19	41	0.95	0	36	22	0.61	-18	0.99	0.02	-15	36	-2.37	-87	0.99	0.71
Panel C: SHBD																		
0%	209	68	0.32	68	0.02	0	466	208	0.44	92	0	0	1014	661	0.65	141	0	0
3%	174	62	0.35	44	0.09	0	320	185	0.58	33	0.03	0	638	489	0.76	1.69	0.01	0
4%	158	60	0.38	34	0.12	0	266	160	0.60	42	0.08	0	545	425	0.78	-23	0.05	0
5%	141	59	0.42	36	0.26	0	222	159	0.71	0.46	0.19	0.02	397	381	0.96	-146	0.17	0.04
6%	124	54	0.43	25	0.36	0	180	137	0.76	-40	0.30	0.03	282	303	1.07	-213	0.28	0.13

(Continue)

TABLE 4: (*CONTINUED*)

	10-Year Horizon						20-Year Horizon						30-Year Horizon					
W/D	**Mean**	**SD**	**CV**	**Min**	**PS**	**PR**	**Mean**	**SD**	**CV**	**Min**	**PS**	**PR**	**Mean**	**SD**	**CV**	**Min**	**PS**	**PR**
Panel D: GLIDE																		
0%	210	64	0.30	90	0.01	0	389	146	0.37	120	0	0	710	348	0.49	201	0	0
3%	163	56	0.34	66	0.08	0	277	136	0.49	46	0.04	0	450	251	0.55	12	0.02	0
4%	148	52	0.35	40	0.17	0	241	137	0.56	35	0.09	0	361	227	0.62	-8.2	0.06	0
5%	136	51	0.37	42	0.23	0	193	111	0.57	5.9	0.18	0	284	233	0.82	-64	0.19	0.03
6%	119	45	0.38	25	0.38	0	147	108	0.73	-46	0.38	0.03	187	217	1.16	-231	0.41	0.16

Notation:

ALL	FTSE All Share index monthly return
BOND	Government Bond index monthly return
SHBD	Portfolio of ALL and BOND
GLIDE	Glide path portfolio of ALL and BOND
W/D	Annual percentage withdrawal rate from savings pool
Mean	Mean end-of-period wealth value
SD	Standard deviation of end-of-period wealth values
CV	Coefficient of variation in end-of-period wealth values (i.e. SD/Mean)
Min	Minimum end-of-period wealth value
PS	Probability of savings pool value falling below £100
PR	Probability of running down the savings pool prior to retirement span, i.e. of financial ruin

from 0.06 to 0.25, and the probability of financial ruin (PR) increases from 0 to 0.09. For all strategies and all horizons, an increase in withdrawal rates does increase the risk of wealth loss. We note that coefficients of variation (CV) remain relatively unchanged as withdrawal rates increase. This implies relatively little material alteration in relationships between reward and standard deviation of expected end-of-horizon wealth.

Given a fixed withdrawal rate, longer investment horizons are generally associated with an increase in mean end-of-period wealth. We note however that standard deviations in expected wealth outcomes increase to a considerably greater extent. CVs therefore also increase with investment horizons. This increase is most pronounced in ALL. For example, at a 4 per cent annual withdrawal, CV increases from 0.80 to 1.74 as the investment horizon extends from ten to thirty years. For all asset classes, regardless of withdrawal rate, CVs increase as holding periods extend. We therefore find no evidence of time diversification benefits, contrary to expectations of the practitioner literature. In contrast, we identify increased risk exposures. We note a similar relationship between length of investment horizon and the relative price of risk if there are no withdrawals (zero withdrawal rates). There is no evidence of time diversification benefits when we assume that returns are reinvested. Compounding will magnify the scale of end-of-period wealth, but variability of wealth outcomes, as indicated by standard deviation, is magnified to a considerably greater extent. Extended time horizons are therefore associated with increased risks to the investor, when risk is measured by a standard deviation of wealth outcomes.

There are, however, further subtle changes in risk exposures as time horizons increase, when risk is measured in terms of potential wealth loss. Probabilities of financial ruin do increase, but probabilities of financial shortfall reduce. With the exception of BOND, we typically find this trend for all investment strategies. For example, with an SHBD strategy and a 6 per cent annual withdrawal rate, the probability of financial shortfall reduces from 0.36 to 0.28 as the investment horizon expands from ten years to thirty years, whereas probability of ruin increases from 0 to 0.13. We therefore identify a form of time diversification benefit, as longer investment horizons are associated with a reduced likelihood that fund value will fall below initial nominal value. The distribution of expected outcomes is such that although the increase in standard deviations is relatively greater than that for expected mean values, the proportion of expected outcomes falling below £100 tends to fall as longer horizons are considered. We examine this issue further in subsequent analyses.

Assuming that life expectancy following retirement is ten years, probability of ruin is 0 for all strategies. Probabilities of financial shortfall are significant however, particularly at high withdrawal rates. Zero likelihood of financial ruin is an attractive outcome, but average UK longevity indicates that most retirees must allow for a considerably longer lifespan. Assuming a twenty-year lifespan, a 6 per cent annual withdrawal rate exposes the retiree to a risk of financial ruin of 0.09 in ALL, whereas BOND exposes the retiree to a 0.02 probability of financial ruin. All strategies that include bonds offer the retiree a greatly reduced exposure to the risk of fund exhaustion, as likelihood of ruin is 0.03 for the SHBD and GLIDE strategies. Probabilities of financial ruin are therefore significantly high for retirees who do not include bonds as part of their investment portfolio.

Assuming a thirty-year horizon, and a 6 per cent annual withdrawal, ALL offers the lowest probability of financial shortfall (0.21), while BOND has the highest, at a 0.99 probability. The retired investor is likely to prefer either a mixed SHBD or a GLIDE strategy to the BOND strategy, as it offers moderate PS risk exposures, even when high annual withdrawals are required.

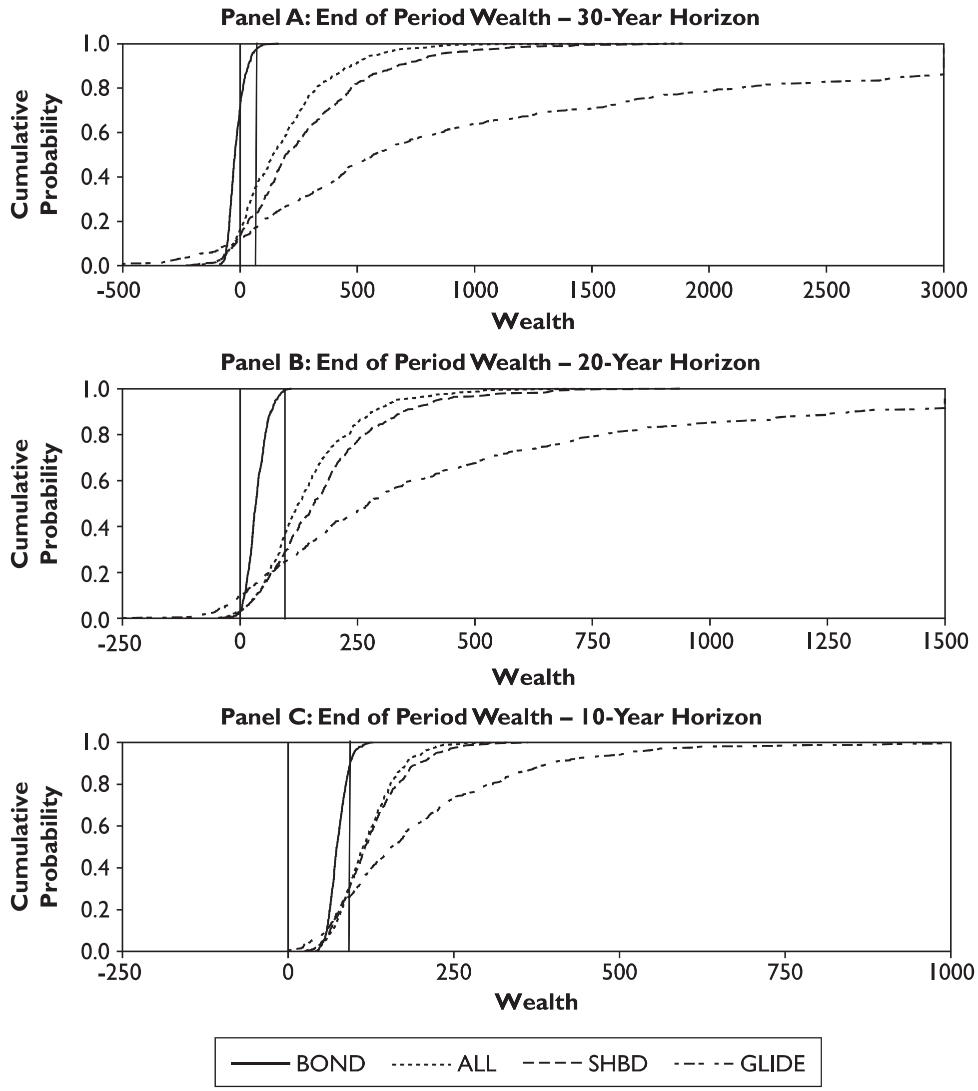
At shorter retirement horizons, an investment in equity becomes a more attractive proposition. If we consider standard deviations or coefficients of variation, it is clear that equity is the most risky investment, but it does offer greater potential capital growth, while limiting probabilities of financial ruin to reasonably acceptable levels. Average life expectancy at retirement is now such that long horizons are likely. Risk to terminal wealth increases with a longer investment horizon, contrary to the common expectation that a longer horizon will confer time diversification benefits.

Cumulative Probability Distributions and End-of-Retirement-Horizon Wealth

In order to provide further insight into the relative performance of each strategy, we plot cumulative probability distributions of end-of-horizon wealth. In Figure 1, we compare strategies over different time horizons. We assume an annual 6 per cent withdrawal. Panels A, B and C respectively illustrate outcomes for thirty-year, twenty-year and ten-year horizons. Cumulative probability at a given wealth level represents the probability of achieving that level or less. This is indicated on the vertical axis. The horizontal axis shows each end-of-period value. Vertical lines identify fund values of £0 and £100, indicating financial ruin and financial shortfall. For example, Panel A indicates that with a thirty-year horizon investment in ALL, the cumulative probability that end value will be £100 or less is 0.21. The probability that value will be more than £100 therefore is 0.79. In contrast, with a BOND investment strategy over thirty years, there is a 0.99 cumulative probability that horizon value will be £100 or less, and therefore only a 0.01 probability it will be more than £100. For any desired wealth outcome, an investment strategy which lies below all others will provide superior performance. Therefore, if an investor wishes to avoid financial shortfall, and a minimum £100 end value is required, equity investment offers best performance, as indicated by a lower cumulative probability than all other strategies. As shown in Panel A, an equity investment actually dominates all strategies if any wealth value exceeding zero is desired. Note that scale values on the horizontal axis differ in each panel, as the range in end-of-period wealth values expands considerably over the longer investment horizons.

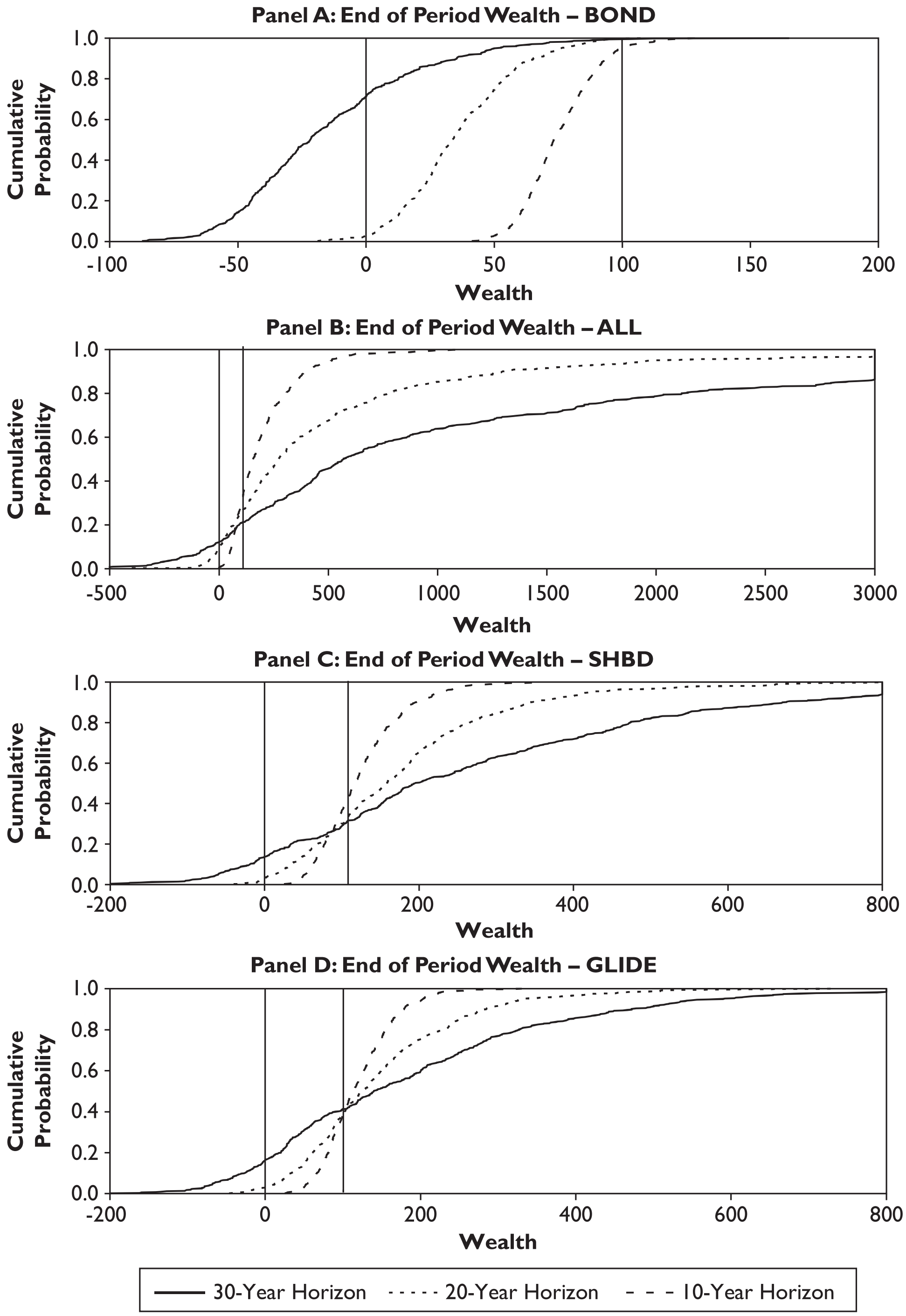
In Panel A, at the thirty-year horizon, the longer left tail of the ALL curve indicates that the likelihood of large wealth losses, signified by negative end-of-period wealth values, is more likely if an ALL investment strategy has been employed. Alternatively, if a retired investor is primarily concerned with upside potential, equity becomes the most attractive investment strategy, as this strategy dominates for most wealth outcomes, as the ALL curve lies below all others. Much of the BOND investment curve lies to the left of a zero end-of-period wealth value. This is because if annual withdrawals of 6 per cent are required, a Bond investment may not generate sufficient annual returns to meet this requirement. If this occurs, fund value will reduce, and may fall to zero or lower. The probability of fund exhaustion (zero wealth level) therefore is highest for BOND. Probability of shortfall also

FIGURE 1: CUMULATIVE PROBABILITY DISTRIBUTIONS AND END-OF-PERIOD WEALTH – COMPARISON OF INVESTMENT HORIZONS



is extremely high. The retiree may consider that one of the mixed investment strategies offers a more attractive prospective outcome. As indicated in Panel A, only a small proportion of either the SHBD or the GLIDE portfolios fall to the left of a zero wealth outcome. In both cases, the probabilities of ruin and of shortfall are at relatively low levels. They also offer relatively good prospects of strong end-of-period wealth. The SHBD strategy curve tends to dominate, as it typically is below the GLIDE curve; this is because a significant investment in shares is maintained throughout, thus maintaining exposure to potentially greater returns. In Panels B and C, a shift of the BOND curve to the right indicates a reduction in the probability of financial ruin, but the probability of shortfall still remains high.

FIGURE 2: CUMULATIVE PROBABILITY DISTRIBUTIONS AND END-OF-PERIOD WEALTH – COMPARISON OF INVESTMENT HORIZONS



In Figure 2 we consider the different strategies in separate panels, and examine the impact of a change in the time horizon on end-of-period wealth, and on the likelihood that this value will reach certain minimum investor expectations. As in Figure 1, scale values on the horizontal axes differ for each investment strategy. In the previous section, we noted longer investment horizons are associated with an increased probability of financial ruin. This trend occurs with all investment strategies. In contrast, we also find that probability of financial shortfall typically declines as time horizons increase. Diagrams for all investment strategies demonstrate changes in risk exposure as retirement horizons extend. In Panel B, we consider ALL as an example. Assuming a ten-year retirement horizon, we observe that there is practically no possibility of financial ruin, but that risk of a shortfall in wealth is approximately 30 per cent. At a twenty-year horizon we note that the risk of financial ruin has increased to slightly below 10 per cent. In contrast, the risk of shortfall has declined to 25 per cent. Dominance of the twenty-year horizon curve over the ten-year curve accentuates for higher levels of end-of-period wealth. For example, the probability that fund value will equal £200 or less at a ten-year horizon is approximately 60 per cent, whereas at a twenty-year horizon it is approximately 40 per cent. At a thirty-year horizon, the risk of financial ruin increases further to approximately 12 per cent, but the risk of a financial shortfall declines again, compared to twenty-year horizon outcome, falling to approximately 21 per cent. This demonstrates that when investing in equities at longer retirement horizons there is an increased risk of suffering extreme losses, but the likelihood of suffering a moderate loss of wealth is reduced. Charts indicating the impact of increasing time horizons on the other investment strategies demonstrate similar changes in risk exposure, although the extent of change tends to be reduced for retirees who have invested greater proportions of their fund in bonds. The GLIDE strategy in Panel D provides a clear example of the presence and absence of time diversification of risk at different wealth levels. For wealth levels of £100 and above, the thirty-year horizon curve dominates the twenty-year and ten-year curves, indicating clear time diversification benefits at those wealth levels. But this benefit pattern is totally reversed for wealth levels of £100 and less, with the ten-year curve dominating the twenty-year curve and the twenty-year curve dominating the thirty-year curve. Panel A is an exception, as in the case of a BOND strategy, the shorter horizon curves always dominate longer horizons. As previously noted, this is because annual withdrawals of 6 per cent probably will exceed returns to a BOND investment, gradually reducing fund value as time horizons extend.

A generally accepted practitioner notion of time diversification is that investment risk diminishes with length of the investment horizon. Our results, however, demonstrate that the concept of risk reduction as horizons change has to be viewed in relation to the magnitude of wealth loss.

CONCLUSIONS

In this paper we examine the risks in retirement finance within the UK context of a retiree who has invested a savings pool in the UK equity and bond markets, and who makes periodic withdrawals for sustenance during the drawdown phase of retirement. Risks are assessed for a range of withdrawal rates, and in terms of several alternative risk measures, and their variations are examined as investment horizons change. We utilised this setting to assess the premise of time diversification of risk in the context of periodic investment withdrawals and the presence of investment compounding effects. It is arguable that our choice of dataset will impact on our conclusions. Any time diversification benefit may be conditional on the level and timing of a major event within an investment time horizon. We draw our data from a fifty-year period that includes the recent global financial crisis, which clearly was a major market correction. Our bootstrap methodology however provides 500 sample runs with randomly selected starting observations for investment returns. This event will therefore appear at different stages in these sample runs, and will be absent from a significant proportion of them. Our computed results from these sample runs provide a distribution of probable outcomes that reflect the likelihood of a future calamitous event, assuming that the previous fifty-year experience is representative of likely future outcomes. We believe this is not an unreasonable assumption.

Our results demonstrate that extreme losses (equivalent to financial ruin) are greater with longer horizons, indicating no benefits of time diversification. But the risk of exposure to moderate losses (equivalent to financial shortfall) does become lower at longer investment horizons. This is consistent with the notion of time diversification benefits. The presence or absence of a time diversification benefit therefore is seen to be conditional on the level and extent of wealth loss. Considering that the average recent retiree can now expect a lifespan of twenty years, or possibly more, they should be aware there is a (small) possibility of financial ruin if higher withdrawal rates are required. We expect that the risk of financial shortfall, although clearly undesirable, will be of secondary importance.

We note that an extended investment horizon is associated with increasing risk exposures for all investment strategies, contrary to the common expectation that an extended investment horizon will confer time diversification benefits, and therefore reduced risk exposures. Regardless of expected withdrawal rates, an expanded investment horizon is associated with increased distributions in expected wealth outcomes when the impact of compounding is included. We find that equity investments warrant consideration, although probabilities of financial ruin become considerably greater, given withdrawal rates in excess of 4 per cent and a reasonably long retirement horizon. An implication of this finding is that a risk-averse retiree should therefore select either a mixed portfolio of shares and bonds (SHBD), or a glide path investment from shares to bonds (GLIDE). With greater life expectancy at retirement, our results imply that the average retiree will include fixed income investments as part of their portfolio. Assuming that higher withdrawal rates are required, they will not impact on overall risk exposure, as indicated by the coefficient of variation, but they do have a material impact on the risk of financial ruin if an equity-only strategy (ALL) is implemented.

The investor with a relatively short time horizon provides an interesting exception. If a lifespan of ten years or less is expected, the probability of financial ruin is zero, regardless of withdrawal rate or investment strategy. An equity portfolio will offer the greatest potential upside. Assuming that a relatively high withdrawal rate also is required, the probability of financial shortfall also is minimal if there is an equity-only strategy. This surprising finding is contrary to the standard advice, as it implies that bonds are not required to manage risk exposure over a shorter horizon. We expect it is highly unlikely that a retiree will experience severe losses over a ten-year horizon.

Our results show that the ruin probabilities of the bootstrapping approach are relatively high. This may be due to the fact that the bootstrap simulation could reproduce a negative skewness that is observed in the actual return distributions. In that respect, the more recent relatively poor market experience is well represented. Whether this experience will persist in the future is a matter of conjecture. Retirement planners using these techniques should be cognisant of these assumptions. Negative skewness has however been a persistent feature of US equity returns, even if the period before 1950 is considered, so there is a reasonable expectation it will impact on future equity returns in many markets. This again implies that risk-averse retirees should hold a mixed investment portfolio.

This study has not taken into consideration taxes and transaction costs such as investment expenses and their effects on the sustainable withdrawal rates. Extensions of this analysis can include these factors, and also consider the inclusion of other asset classes and further investment/withdrawal strategies. This will become possible as extended datasets for other asset types and for separate sectors of the equity markets become available.

We also note the unexpected outcome that an extended investment horizon is associated with increasing risk exposures. The finding is contrary to the common expectation that an extended investment horizon will confer time diversification benefits, and therefore reduced risk exposures. Regardless of expected withdrawal rates, an expanded investment horizon is associated with increased distributions in expected wealth outcomes, when the impact of compounding is included. We believe it is likely that this outcome will be repeated under most future circumstances, although not all.

ENDNOTES

1. See Office for National Statistics (2015), Statistical Bulletin, 'Life Expectancy at Birth and at Age 65, by Local Areas in England and Wales, 2012–2014', www.ons.gov.uk/peoplepopulationandcommunity/birthsdeathsandmarriages/lifeexpectancies/bulletins/lifeexpectancyatbirthandatage65bylocalareasinenglandandwales/2015-11-04.
2. A detailed description of the bootstrap re-sampling technique can be found in Efron (1979).
3. This data were supplied by Thomson Datastream.
4. The source for the long bond yield series is the Bank of England.
5. With thanks to Shane Whelan for this suggestion. Full details regarding the estimation of monthly returns from long bond yield data are available from the authors.
6. Coefficient of variation is defined as the standard deviation divided by the mean.

REFERENCES

Alles, L. and Murray, L. (2009). Investment Performance and Holding Periods: An Investigation of the Major UK Asset Classes, *Journal of Asset Management*, Vol. 10, No. 5, pp. 280–293.
Arnott, R. (2004). Sustainable Spending in a Lower Return World, *Financial Analysts Journal,* Vol. 60, No. 5, pp. 6–9.
Basu, A., Byrne, A. and Drew, M. (2011). Dynamic Lifecycle Strategies for Target Date Retirement Funds, *Journal of Portfolio Management,* Vol. 37, No. 2, pp. 83–96.
Bengen, W. (1994). Determining Withdrawal Rates Using Historical Data, *Journal of Financial Planning,* Vol. 7, No. 4, pp. 171–180.
Efron, B. (1979). Bootstrap Methodology: Another Look at the Jackknife, *Annals of Statistics,* Vol. 7, No. 13, pp. 1–26.
Hickman, K.Y., Hunter, H., Byrd, J., Beck, J. and Tempering, W. (2001). Life Cycle Investing, Holding Periods and Risk, *Journal of Portfolio Management*, Vol. 27, No. 2, pp. 101–111.
Kritzman, M. (1994). What Practitioners Need to Know about Time Diversification, *Financial Analysts Journal,* Vol. 50, No. 1, pp. 14–18.
Milevsky, M. and Robinson, C. (2005). A Sustainable Spending Rate without Simulation, *Financial Analysts Journal,* Vol. 61, No. 6, pp. 89–100.
Pye, G.B. (2000). Sustainable Investment Withdrawals, *Journal of Portfolio Management,* Vol. 26, No. 4, pp. 73–83.
Samuelson, P. (1969). Lifetime Portfolio Selection by Dynamic Stochastic Programming, *Review of Economics and Statistics,* Vol. 51, No. 3, pp. 239–246.